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2- Brown ; J. D. , Music and Mathematics since the Seventeenth Century , Mathematics Teacher, Dec. 1968 ,1968 , PP783-87
3- Botts ;Truman , More on the Mathematics of Musical Scales ,Mathematics Teacher , Jan. 1974,PP. 75-84
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6- Haak ;Sheila , Using The Monochord: A Classroom Demonstration on the Mathematics of Musical Scales , Mathematics Teacher , March 1982 , PP. 239 - 244
7- Lawlis;F., The Music.. Mathematics ,Mathematics Teacher, vol. 60,1967,PP 593-596
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10- Peak ;Philip , Have You Read ?, Mathematics Teacher ,May ,1967,P. 504
11- Rice ;Jack , The Affinity of Mathematics to Music , Mathematics Teacher , March 1986 ,PP. 269-271
12- Ridout ;Theodore , Sebastian and the "Wolf" , Mathematics Teacher , Feb. ,1955,P. 84-86

```
1940 IF CC>12 THEN FF=FF+2*(R+DV)
1950 CU=CU+1:IF CU<12 THEN FG=0
1960 IF CU>12 AND CC<25 THEN FF=2*(R+DV)+35:FG=6
1970 IF CU>24 AND CC<37 THEN FF=4*(R+DV)+65:FG=12
1980 IF CU>36 AND CC<48 THEN FF=6*(R+DV)+100:FG=18
1990 NM=CC:PX=(CC MOD 12) +1
2000 IF CU<13 THEN FF=3
2010 LINE(CX+C-R-DV,CY-R-DV+FF)-(CX+C+R+DV,CY+R+DV+FF),PX+1,B
2020 IF CC =0  THEN GOSUB 1340
2030 IF CC =12 THEN GOSUB 1420
2040 IF CC =24 THEN GOSUB 1580
2050 IF CC =36 THEN GOSUB 1500
2060 H=FG+6
2070 G1=5+ABS(C/8)MOD 79::LOCATE H,G1-1:COLOR HY+2 :PRINT (12-S(CL)) MOD
12;:HJ=H
2080 FOR B=0 TO 11
2090 SOUND 37+B+CC,1:D=B+1 :FD =CC MOD 12
2100 XB=X(D)+C:YB=Y(D)+FF:XH=C+XX(D):H=SS(FD,B)
2101 IF GRO$="O" OR GRO$="o" THEN HH=H:GOSUB 3100 ELSE 2145
2102 H=HH
2143 GOTO 2150
2145 IF H=0 THEN H=12
2150 IF B=0 THEN H1=SS(FD,B):LK=52/12 ELSE H1=SS(FD,B-1):LK=0
2155 IF GRO$="O" OR GRO$="o" THEN HH=H1:GOSUB 3100 ELSE 2160
2156 H1=HH
2157 GOTO 2170
2160 IF H1=0 THEN H1=12
2170 LINE (XB,YB+6 )-(XB,YB+11),H+1
2180 LINE (CX+C-R-DV-1,YB-4*H+5)-(CX+C-R-DV,YB-4*H+5),H+1
2190 IF GRH$="1" THEN 2200 ELSE 2210
2200 LINE (XB,YB+5)-(XH,YB-H*4+5),H+1,BF:GOTO 2230
2210 PSET(XB-2,YB-YY(H1)+10),H+1
2220 LINE (XB-2,YB-YY(H1)+10)-(XH-LK-2,YB-YY(H)+10),H+1
2230 NEXT B
2240 GOSUB 1660:LOCATE 25,15:PRINT "PLAYING THE CHAIN NO. ";CL+1;" IN ";N$;"
: THE CHAIN (";TY$;")";:PLAY TY$ :TY$="":LOCATE 25,15:PRINT SPC(64);
2250 G1=5+ABS(C/8)MOD 79::LOCATE HJ-5,G1-1:COLOR HY+2 :PRINT S(CL);
2260 NEXT CL
2270 NEXT KIND
2280 IF CU>47 THEN GOTO 2300
2290 IF CC<8 THEN 1900 ELSE 1890
2300 LOCATE 18 ,1:END
3100 REM oshia modification
3110 IF N$="O" AND HH=0 THEN HH=0
3120 IF N$="R" AND HH=0 THEN HH=0
3130 IF N$="RI" AND HH=0 THEN HH=12
3140 IF N$="I" AND HH=0 THEN HH=12
3150 RETURN
```

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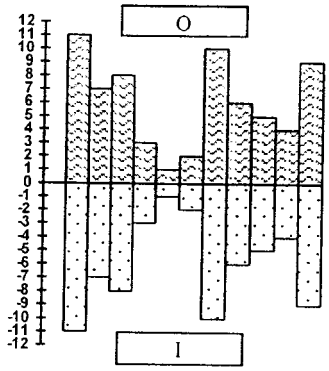
1- O'Shea;T. , Geometric Transformation and Musical Composition , Mathematics Teacher , Oct. 1979,PP.523-528

2- Maor;Eli, What is so Mathematical About Music ? , Mathematics Teacher, Sep. 1979 , PP. 414-422

```
1390 SS$(I,J)=B$(I+2,J+2)
1400 NEXT J,I
1410 RETURN
1420 N$="R":LOCATE 7,1:PRINT N$
1430 FOR I=0 TO 11
1440 S(I)=A(13-I,1)
1450 FOR J=0  TO 11
1460 SS(I,J)=A(I+2,13-J)
1470 SS$(I,J)=B$(I+2,13-J)
1480 NEXT J,I
1490 RETURN
1500 N$="RI":LOCATE 19,1:PRINT N$
1510 FOR I=0 TO 11
1520 S(I)=A(13-I,1)
1530 FOR J=0  TO 11
1540 SS(J,I)=A(13-I,J+2)
1550 SS$(J,I)=B$(13-I,J+2)
1560 NEXT J,I
1570 RETURN
1580 N$="I":LOCATE 13,1:PRINT N$
1590 FOR I=0 TO 11
1600 S(I)=A(2+I,1)
1610 FOR J=0  TO 11
1620 SS(J,I)=A(I+2,2+J)
1630 SS$(J,I)=B$(I+2,2+J)
1640 NEXT J,I
1650 RETURN
1660 FOR I=0 TO 11
1670 TY$=SS$(I,FD)+TY$
1680 NEXT I
1690 RETURN
1700 N$="O":LOCATE 1,1:PRINT N$
1710 B1=B
1720 B2=SS(CL,B )
1730 HP=B2
1740 RETURN
1750 B1=SS(CL,B)
1760 B2=SS(CL,HH)
1770 RETURN
1780 REM
1790 PRINT N:M=N
1800 FF=0:DV=0:CX=0:CY=0:FG=0:C=0:R=0:CU=0:N$="O"
1810 FOR I=0 TO 11:EL$=TONE$(2,I):S(I)=VAL(EL$):NEXT
1820 B=8*ATN(1)/N
1830 R=23:DV=3:CX=R+5:CY=R+2*DV+10
1840 FOR I=1 TO 12
1850 X(I)=CX+(52/12)*I-30:Y(I)=2*R+13
1860 XX(I)=CX+(52/12)*(I+1)-30:YY(I)=4*I+5
1870 NEXT I
1880 CLS :
1890 IF CC>12 THEN FF=FF+2*(R+DV)
1900 CC=-1:GOSUB 1340
1910 FOR KIND = 1 TO 4
1920 FOR CL=0 TO 11:CC=CC+1
1930 C=((CC MOD 12))*2*(R +DV)+10
```

```
930 NM=CC:PX=(CC MOD 12) +1
940 IF CU<13 THEN FF=3
950 LINE(CX+C-R-DV,CY-R-DV+FF)-(CX+C+R+DV,CY+R+DV+FF),PX+1,B
960 BB=8*ATN(1)/360
970 IF CC =0  THEN GOSUB 1340
980 IF CC =12 THEN GOSUB 1420
990 IF CC =24 THEN GOSUB 1580
1000 IF CC =36 THEN GOSUB 1500
1010 H=FG+6  :G1=5+ABS(C/8)MOD 79::LOCATE H,G1-1:COLOR HY+2 :PRINT
B$(1,CL);
1020 FOR B=0 TO 11
1030 G1=5+ABS(C/8)MOD 79::LOCATE H,G1-1:COLOR HY+2 :PRINT (12-S(CL)) MOD
12;:HJ=H
1040 HH=(B+1) MOD 12: GOSUB 1750:SOUND 37+B+CC,1
1050 XB=X(B1)+C:YB=Y(B1)+FF:XH=C+X(B2):YH=FF+Y(B2):GOTO 1060
1060 LINE (XB,YB)-(XH,YH),B+1
1070 NEXT B
1080 GOSUB 1660:LOCATE 25,15:PRINT "PLAYING THE CHAIN NO. ";CL+1;" IN ";N$;"
: THE CHAIN (";TY$;")";:PLAY TY$ :TY$="":LOCATE 25,15:PRINT SPC(64);
1090 G1=5+ABS(C/8)MOD 79::LOCATE HJ-5,G1-1:COLOR HY+2 :PRINT S(CL);
1100 NEXT CL
1110 NEXT RW
1120 IF CU>47 THEN GOTO 1140
1130 IF CC<8 THEN 830 ELSE 840
1140 LOCATE 25 ,1:PRINT "type y for bars graph and n for end";:INPUT ANS$
1150 IF ANS$="Y" OR ANS$="y" THEN 1160 ELSE LOCATE 23,1:END
1160 LOCATE 25 ,1:PRINT SPC(60);:LOCATE 25,1:PRINT "type 1 for bars grah or 2 for
jagged graph";:INPUT GRH$
1162 LOCATE 25 ,1:PRINT SPC(60);:LOCATE 25,1:PRINT "type O for OSHEA
Modification or N for normal graph";:INPUT GRO$:GOTO 1780
1170 IF TONE <0 OR TONE>11 THEN BEEP:GOTO 270
1180 IF I=0 THEN START=TONE :FOR P=0 TO 11 :SS=MT(1,P):MT(2,P)=(TONE+SS)
MOD 12:NEXT:GOSUB 1250
1190 T(I)=TONE
1200 FOR X=0 TO I-1
1210 IF T(X)=TONE THEN BEEP:BEEP:BB$="n" :GOTO 270
1220 NEXT X
1230 BB$="y"
1240 RETURN
1250 FOR U=0 TO 11 :PH=(MT(2,U)+START) MOD
12:PH$=STR$(PH):M$(3,U)=PH$:NEXT :RETURN
1260 FOR U=1 TO 13
1270 FOR V=1 TO 13
1280 FOR Z=1 TO 12
1290 EL$=TONE$(2,Z-1):IF VAL(EL$)=A(V,U) THEN B$(U,V)=TONE$(1,Z-1):GOTO
1310
1300 NEXT Z
1310 NEXT V
1320 NEXT U
1330 RETURN
1340 N$="O":LOCATE 1,1:PRINT N$
1350 FOR I=0 TO 11
1360 S(I)=A(I+2,1)
1370 FOR J=0  TO 11
1380 SS(I,J)=A(I+2,J+2)
```

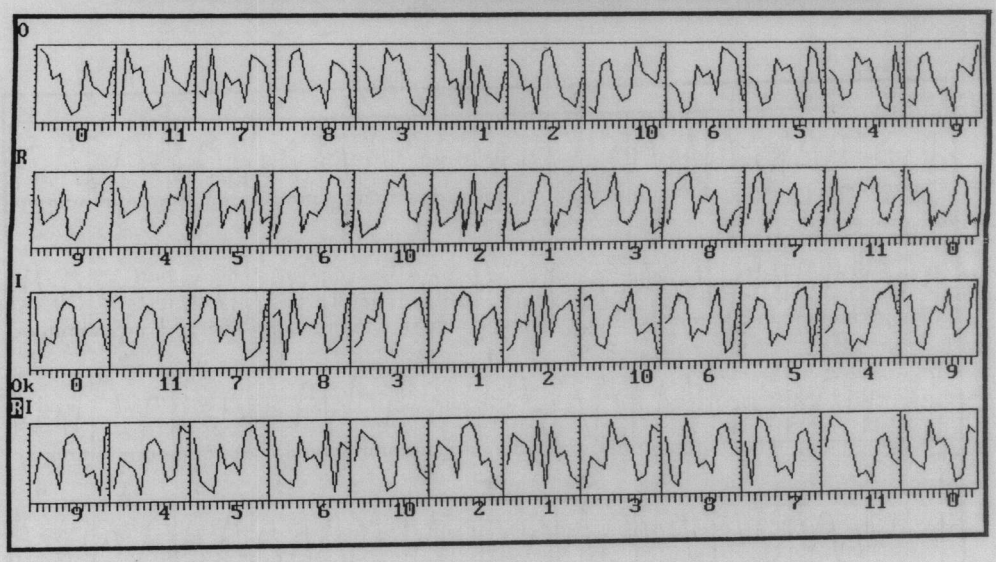
```
400 K=N
410 L=13
420 REM
430 FOR I=1 TO 12:EL$=TONE$(2,I-1):B$(1,12-I+1)=TONE$(1,I-1):B$(I+1,1)=TONE$(1,I-
1):A(1,I+1)=VAL(EL$):A(I+1,1)=(12-VAL(EL$)) MOD 12:NEXT
440 REM
450 FOR J=2 TO L :FOR S=2 TO L
460 A(S,J)=(A(1,J)+A(S,1)) MOD 12
470 NEXT S
480 NEXT J
490 GOSUB 1260 :GOSUB 1350
500 FOR J=1 TO L:FOR S=1 TO L
510 LOCATE J,3*S-2:PRINT A(J,S)
520 NEXT S:NEXT J
530 CX=3*8*L+13:CY=1*14*L+8
540 LOCATE 1,2:PRINT "*"
550 FOR X=0 TO CX STEP 24
560 LINE (X,0)-(X,CY-8),5:NEXT X
570 FOR Y=0 TO CY STEP 14
580 LINE (0,Y)-(CX,Y),5:NEXT Y
590 FOR I=1 TO L:FOR J=1 TO L
600 AIJ$=B$(I,J)
610 LOCATE J,38+3*I:PRINT AIJ$
620 NEXT J:NEXT I
630 CX=3*8*2*L+16+315:CY=1*14*L+8
640 LOCATE 1,41:PRINT "*"
650 FOR X=316 TO CX-16 STEP 24
660 LINE (X,0)-(X,CY-8),3:NEXT X
670 FOR Y=0 TO CY STEP 14
680 LINE (316,Y)-(CX-328,Y),3:NEXT Y
690 LOCATE 20,1:INPUT "do you want graph (Y/N)",ANS$
700 IF ANS$="y" OR ANS$="Y" THEN GOTO 730
710 IF ANS$="n" OR ANS$="N" THEN END
720 IF ANS$<>="n" OR ANS$="N" OR ANS$= "y" OR ANS$="Y"  THEN BEEP :GOTO
690
730 COLOR 7,0
740 REM
750 PRINT N:M=N
760 FOR I=0 TO 11:EL$=TONE$(2,I):S(I)=VAL(EL$):NEXT
770 B=8*ATN(1)/N:DIM X(N),Y(N),XX(M),YY(M)
780 R=23:DV=3:CX=R+5:CY=R+2*DV+10
790 FOR I=0 TO 11
800 XI=CX+R*COS(I*B):YI=CY+R*SIN(I*B):X(I)=XI:Y(I)=YI
810 NEXT I
820 CLS :CC=-1
830 FOR RW=1 TO 4
840 FOR CL=0 TO 11
850 CC=CC+1
860 C=((CC MOD 12))*2*(R +DV)
870 CU=CU+1
880 IF CC>12 THEN FF=FF+2*(R+DV)
890 IF CU<12 THEN FG=0
900 IF CU>12 AND CC<25 THEN FF=2*(R+DV)+35:FG=6
910 IF CU>24 AND CC<37 THEN FF=4*(R+DV)+65:FG=12
920 IF CU>36 AND CC<48 THEN FF=6*(R+DV)+100:FG=18
```

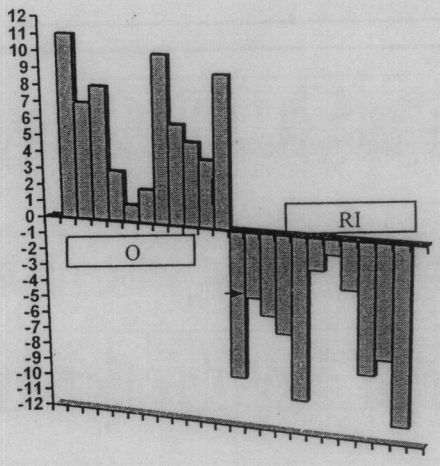


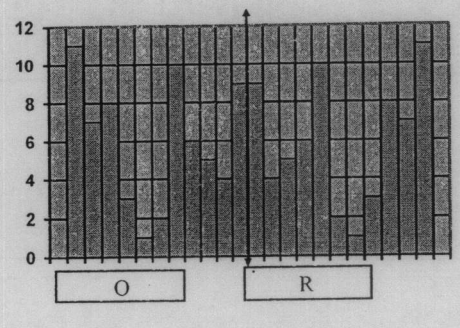
The following program draws the circular representation of all 48 chains and has the following features:

- Compute all elements of group modulo 12 and print them in table.
- Print the corresponding tone table.
- Draw all the 48 chains in circular representation.
- Play all the 48 chains.

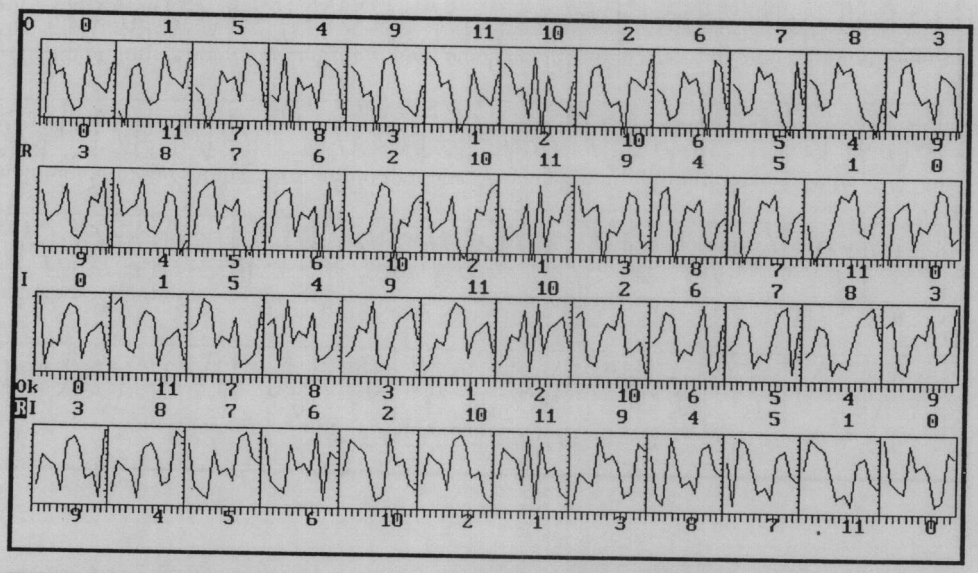
```
100 REM Dr.\ Mahmoud Ibrahim Badr
105 REM Group Theory and Musics graphs
110 DIM V(2,12),Z(2,12),S(12)
120 CLS :DIM TONE$(4,12),T(12),B$(13,50)
130 N=12
140 DIM M$(3,12),MT(3,12),A(13,13),SS(12,12),SS$(12,12)
150 M$(0,0)="C":M$(0,1)="Db":M$(0,2)="D":M$(0,3)="Eb":M$(0,4)="E":M$(0,5)="F"
160 M$(0,6)="Gb":M$(0,7)="G":M$(0,8)="Ab":M$(0,9)="A":M$(0,10)="Bb":M$(0,11)="B"
170 M$(1,0)="C":M$(1,1)="C#":M$(1,2)="D":M$(1,3)="D#":M$(1,4)="E":M$(1,5)="F"
180 M$(1,6)="F#":M$(1,7)="G":M$(1,8)="G#":M$(1,9)="A":M$(1,10)="A#":M$(1,11)="B"
190 M$(2,0)="0":M$(2,1)="1":M$(2,2)="2":M$(2,3)="3":M$(2,4)="4":M$(2,5)="5"
200 M$(2,6)="6":M$(2,7)="7":M$(2,8)="8":M$(2,9)="9":M$(2,10)="10":M$(2,11)="11"
210 FOR I=0 TO 11 :LOCATE 1,6*I+5:PRINT M$(0,I):NEXT
220 FOR I=0 TO 11 :LOCATE 2,6*I+5:PRINT M$(1,I):NEXT
230 FOR I=0 TO 11 :MT(1,I)=I:LOCATE 3,6*I+5:PRINT M$(2,I):NEXT
240 LOCATE 7,1:PRINT SPC(70):LOCATE 7,1 :INPUT "Enter (0) b or (1) #";KIND
250 IF KIND=0 OR KIND=1 THEN 260 ELSE BEEP:BEEP:GOTO 240
260 FOR I=0 TO 11
270 BB$="y":LOCATE 7,1:PRINT SPC(77):LOCATE 7,1:INPUT "Enter the number of which
refere to tones",T$:TONE=VAL(T$)
280 GOSUB 1170
290 IF T$=" " THEN 270
300 IF BB$="n" THEN 270
310 TONE$(1,I)=M$(KIND,TONE)
320 TONE$(2,I)=M$(3,TONE)
330 LOCATE 5,6*I+5:PRINT TONE$(1,I):LOCATE 6,6*I+5:PRINT TONE$(2,I)
340 NEXT I
350 FOR I=0 TO 11 :LOCATE 14,6*I+5:PRINT TONE$(1,I):NEXT
360 FOR I=0 TO 11 :LOCATE 15,6*I+5:PRINT TONE$(2,I):NEXT
370 INPUT O
380 REM 6/4/ 1988
390 CLS:KEY OFF:SCREEN 9:LOCATE ,,0
```

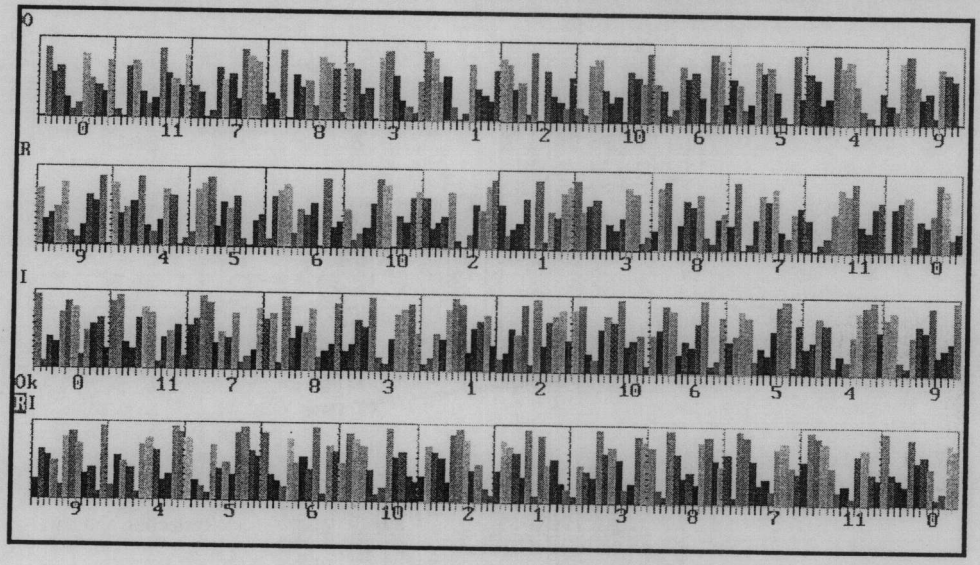




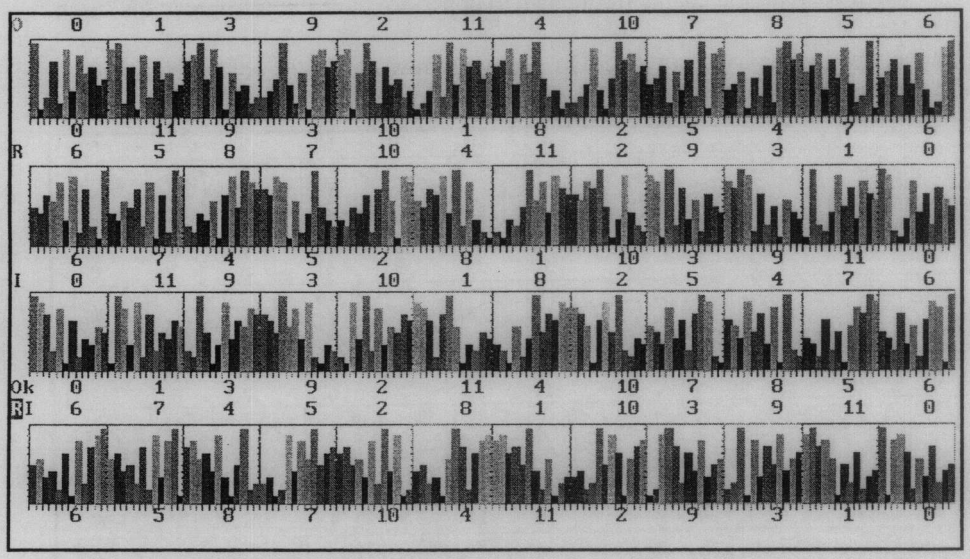


Reflection about M


O 0 1 5 4 9 11 10 2 6 7 8 3
R 3 8 7 6 2 10 11 9 4 5 1 0
I 0 1 5 4 9 11 10 2 6 7 8 3
Ok 0 11 7 8 3 1 2 10 6 5 4 9
RI 3 8 7 6 2 10 11 9 4 5 1 0


O
R
I
Ok
RI

The graphs of first example are:



We can conclude that:

In the circular representation:

- We can deduce every element the set (O) by rotation equal to 30 * inverse of the element (upper graph numbers), and the direction of drawing is clockwise.
- The retrograde elements represent the same shape of (O), but different only in the color of element and the direction of drawing which is anti-clockwise.
- The I shapes are horizontal reflection of the O shapes.
- The RI shapes are similar to I shapes but different in the color of the elements, and the direction of drawing is anti-clockwise.

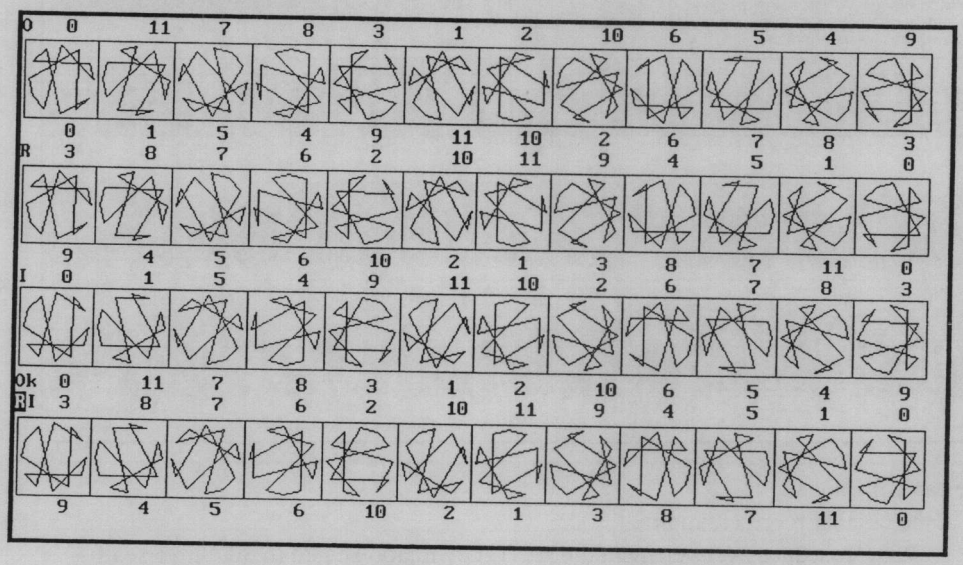
And similar to O'Shea graphics we have:

- The O element can be deduced from the original chain, by transform its element an inverse of elements, but we tack into account addition of element and its element with respect to modulo 12.
- The R elements can be deduced as inverse about middle element of the corresponding element of O. O'Shea also described the retrograde as vertical reflection of the correspondence O chain.
- The I elements he can be described as glide – reflection or horizontal reflection, but O'Shea change numbers to its minus reverses and change 0 to –12 in this case
- The RI elements are the reverse order of the O shapes. O'Shea described the retrograde –inversion as clockwise rotation by 180 about last point of the chain, but O'Shea also change numbers to its minus reverses and change 0 to –12 in this case.

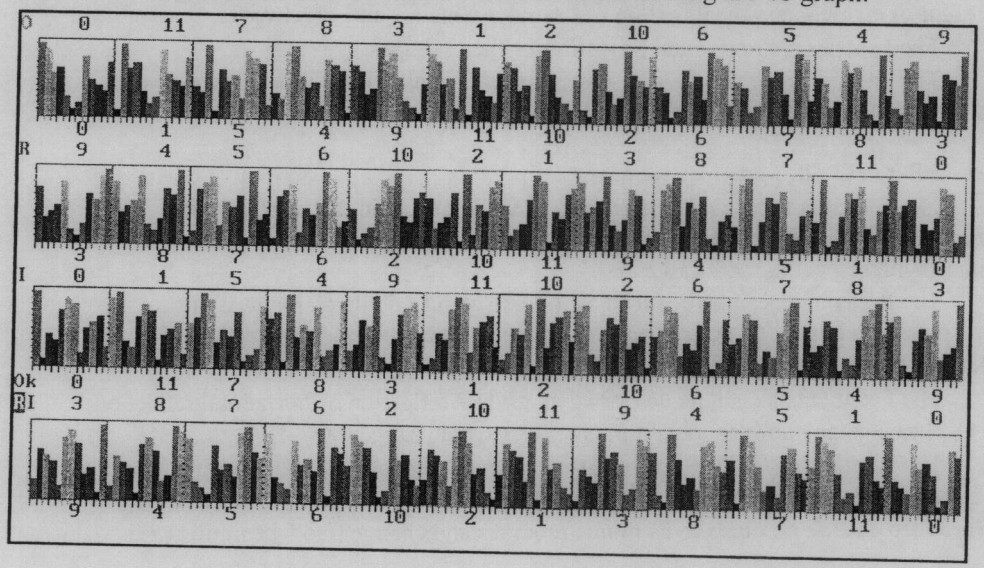
We can connect the two styles by the change of direction corresponds to rotation 180 about the last point of the chain, the shift in each set is corresponds to rotation.

- When we read the rows from left to right, we have the Original chains.
- When we read the rows from right to left, we have the retrograde chains.
- When we read the Column from upper to down, we have the inversion chains.
- When we read the chains from down to upper, we have the retrograde inversion chains.

From the table we have 12 rows for O , 12 reverse row for R , 12 column for inversion and 12 inverted column for RI



O'Shea was discussed in mathematics teacher the second example by using the graph, which represent each tone with its number. The following are 48 graph:



C B A Eb Bb Db Ab D F E G Gb

Now we begin the numerating from the tone Do or C Or adding inverse of 4 (which E number), the we have:

C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B
C	C#	D	D#	E	F	F#	G	G#	A	A#	B
0	1	2	3	4	5	6	7	8	9	10	11

Now we replace the tone by its numbers in the original chain, then we have:

C	B	A	Eb	Bb	Db	Ab	D	F	E	G	Gb
0	11	7	8	3	1	2	10	6	5	4	9

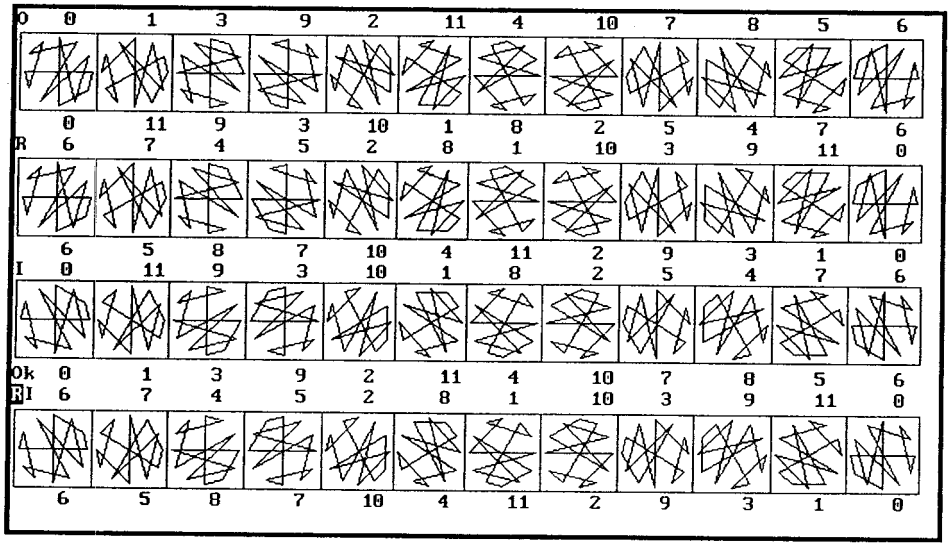
We now construct the table, which based on group theory as follow:

1- We put the numbers of chain in first row.
2- And put its reverse in first column.
3- And compute the addition operation modulo 12 for the table.

		O ⇒											⇐ R
I		0	11	7	8	3	1	2	10	6	5	4	9
⇓	0	0	11	7	8	3	1	2	10	6	5	4	9
	1	1	0	8	9	4	2	3	11	7	6	5	10
	5	5	4	0	1	8	6	7	3	11	10	9	2
	4	4	3	11	0	7	5	6	2	10	9	8	1
	9	9	8	4	5	0	10	11	7	3	2	1	6
	11	11	10	6	7	2	0	1	9	5	4	3	8
	10	10	9	5	6	1	11	0	8	4	3	2	7
	2	2	1	9	10	5	3	4	0	8	7	6	11
	6	6	5	1	2	9	7	8	4	0	11	10	3
	7	7	6	2	3	10	8	9	5	1	0	11	4
⇑	8	8	7	3	4	11	9	10	6	2	1	0	5
RI	3	3	2	10	11	6	4	5	1	9	8	7	0

And by replacing each number and the corresponding tone we have:

	O ⇒						⇐ R					
I	C	B	G	Ab	Eb	Db	E	Bb	Gb	F	E	A
⇓	Db	C	Ab	A	E	D	Eb	B	B	G	Gb	Bb
	F	E	C	Db	Ab	Gb	G	Eb	B	Bb	A	D
	E	Eb	B	C	G	F	Gb	D	Bb	A	Ab	Db
	A	Ab	E	F	C	Bb	B	G	Eb	D	Db	Gb
	B	Bb	Gb	G	D	C	Db	A	F	E	Eb	Ab
	Bb	A	F	Gb	Db	B	C	Ab	E	Eb	D	G
	D	Db	A	Bb	F	Eb	E	C	Ab	G	Gb	B
	Gb	F	Db	D	A	G	Ab	E	C	B	Bb	Eb
	G	Gb	D	Eb	Bb	Ab	A	F	Db	C	B	E
⇑	Ab	G	Eb	E	B	A	Bb	Gb	D	Db	C	F
RI	Eb	D	Bb	B	Gb	E	F	Db	A	Ab	G	C



	O ⇒											⇐ R
I	E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Bb
⇓	Eb	E	Gb	C	F	D	G	Db	Bb	B	Ab	A
	Db	D	E	Bb	Eb	C	F	B	Ab	A	Gb	G
	G	Ab	Bb	E	A	Gb	B	F	D	Eb	C	Db
	D	Eb	F	B	E	Db	Gb	C	A	Bb	G	Ab
	F	Gb	Ab	D	G	E	A	Eb	C	Db	Bb	B
	C	Db	Eb	A	D	B	E	Bb	G	Ab	F	Gb
	Gb	G	A	Eb	Ab	F	Bb	E	Db	D	B	C
	A	Bb	C	Gb	B	Ab	Db	G	E	F	D	Eb
	Ab	A	B	F	Bb	G	C	Gb	Eb	E	Db	D
⇑	B	C	D	Ab	Db	Bb	Eb	A	Gb	G	E	F
RI	Bb	B	Db	G	C	A	D	Ab	F	Gb	Eb	E

- When we read the rows from left to right, we have the Original chains.
- When we read the rows from right to left, we have the retrograde chains.
- When we read the Column from upper to down, we have the inversion chains.
- When we read the chains from down to upper, we have the retrograde inversion chains.

Now we represent each chain on circle, such that we put each number (or tone) from 0 to 11 clockwise and connect each number (or tone) by the number (or tone) which follow it, we have the following graphics:

Example (2)

Using the technique of group theory deduce the all chains of the following chain and represent them on circle.

C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B
C	C#	D	D#	E	F	F#	G	G#	A	A#	B
0	1	2	3	4	5	6	7	8	9	10	11

Now we begin the numerating from the tone Me or E Or adding inverse of 4 (which E number), then we have:

C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B
C	C#	D	D#	E	F	F#	G	G#	A	A#	B
8	9	10	11	0	1	2	3	4	5	6	7

Now we replace the tone by its numbers in the original chain, then we have

E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Bb
0	1	3	9	2	11	4	10	7	8	5	6

We now construct the table, which based on group theory as follow:

1- We put the numbers of chain in first row.
2- And put its inverses in first column.
3- And compute the addition with modulo 12.

$\oplus_{12}$	0	1	3	9	2	11	4	10	7	8	5	6
0	0	1	3	9	2	11	4	10	7	8	5	6
11	11	0	2	8	1	10	3	9	6	7	4	5
9	9	10	0	6	11	8	1	7	4	5	2	3
3	3	4	6	0	5	2	7	1	10	11	8	9
10	10	11	1	7	0	9	2	8	5	6	3	4
1	1	2	4	10	3	0	5	11	8	9	6	7
8	8	9	11	5	10	7	0	6	3	4	1	2
2	2	3	5	11	4	1	6	0	9	10	7	8
5	5	6	8	2	7	4	9	3	0	1	10	11
4	4	5	7	1	6	3	8	2	11	0	9	10
7	7	8	10	4	9	6	11	5	2	3	0	1
6	6	7	9	3	8	5	10	4	1	2	11	0

Now we remove the first row and first column, and read the chains as the arrows refer.

	O ⇒									⇐ R		
I	0	1	3	9	2	11	4	10	7	8	5	6
⇓	11	0	2	8	1	10	3	9	6	7	4	5
	9	10	0	6	11	8	1	7	4	5	2	3
	3	4	6	0	5	2	7	1	10	11	8	9
	10	11	1	7	0	9	2	8	5	6	3	4
	1	2	4	10	3	0	5	11	8	9	6	7
	8	9	11	5	10	7	0	6	3	4	1	2
	2	3	5	11	4	1	6	0	9	10	7	8
	5	6	8	2	7	4	9	3	0	1	10	11
	4	5	7	1	6	3	8	2	11	0	9	10
⇑	7	8	10	4	9	6	11	5	2	3	0	1
RI	6	7	9	3	8	5	10	4	1	2	11	0

And by replace numbers by its corresponding tone, then we have the following table:

Group Theory and Schoenberg Music

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Group theory is one of the most important topics in mathematics. It has wide application in Physics and crystals. One of its applications was in music, where Schoenberg used the technique of group in his music. In 1908 Schoenberg was searched the atonality music. In his searching about unifying theory, he was put the basis of twelve-tone technique as follow:

1- We order the twelve-tones as composer appreciate without any tonality base and this called the Original chain.
2- We read the array from right to left direction and called this as retrograde chain.
3- We begin from the start with inverting and called this as Inverted chain.
4- We read the inverted chain in reverse order and called this as Retrograde Inverted.

The idea of tonality is removing the important of any tone with respect to the other tones.

Schoenberg was discovered 48 permutation of ordinal tones , and he was begin to define 4 permutation of original chain:

O , RO , IO , (RI)O
Where O = (x0,x1,x2,..... , x11)
R O =(x11,x10, ...,x0)
I O =(-x0,-x1,.....,-x11)
(RI)O =(-x11,-x10,.....,-x0)
(RI)O =(-x11,-x10,.....,-x0)

In Schoenberg's technique we see the important of modulo 12, we can regard R, RI and I as operators, then this lead us to group theory.

There is another important feature that was known by Transpositions, which can be illustrate as follows:

Let t is any integer, and X is one of the four operators O,RO,IO,RIO , we define T X as adding the number t modulo 12 to the tone X . It clear that, there is 12 Transpositions in each set , then we have 48 chains, but some of them equivalent and this explain the repetition in musical composition.

Example (1):

Using the technique of group theory deduce the all chains of the following chain, and represent them on a circle.

E	F	G	Db	Gb	Eb	Ab	D	B	C	A	Eb

We know that the true order of tones is: