Design Principles of Digital Resources to Foster Creative Mathematical Thinking Affordances: The Case of "Math for Biology" Resource

BY

Nataly Essonnier
IREM-Grenoble Institute for Research on Mathematics Education – France
nataly.essonnier@formation-industries-savoie.fr

Mohamed El-Demerdash
Menoufia University- Faculty of Education
m_eldemerdash70@edu.menofia.edu.eg

Abstract

This article is based on our work and experience developed within the framework of a European project MC squared (MC2) (http://mc2-project.eu/). We have studied the collaborative design of innovative digital resources. These resources, called “c-books” (c for creative), were produced within a socio-technological environment under development by an education community acting designer, viewed as a Community of Interest (CoI), bringing together mathematics teachers, computer scientists and researchers in mathematics and in mathematics education. This article first discusses creativity and outlines the conceptualization Creative Mathematical Thinking (CMT) adopted within the MC2 project. Then, we present a case study: the design of the resource entitled “Math for Biology” which includes many examples of mathematical modeling in a biological context and discusses the design choices to provide as many affordances as possible to promote CMT in its end users. The evaluation of the CMT affordances of this resource has been possible through a conceptual tool; the CMT assessment grid, based on an agreement Likert scale. The assessment of the CMT affordances was done a priori by three designers who did not participate in the design process. Finally, we bring to the fore design principles which emerged during the design process in relation to the CMT affordances of the resource produced.

Keywords

Mathematics Education, Community of Interest, Creativity, Creative Mathematical Thinking, Socio-technological Environment, Biomathematics, Mathematical Modeling.

1. Introduction

Promoting innovation skills and creativity is a central purpose of P21's Partnership of the Framework for 21st Century Learning (2011, p. 3). Learning and innovation skills are increasingly recognized as those needed for an increasingly complex life and work environments in the 21st century. A focus on creativity, critical thinking, communication, and collaboration is thus essential to prepare students for the future. Likewise, the European Union (EU) considers "creativity, innovation and risk-taking" as part of the key competencies for lifelong learning aiming at personal and social empowerment for EU citizens (EC, 2006).

In general, CMT is seen as an individual and collective construction of mathematical meanings, norms and uses in novel and useful ways (Sternberg, 2003). Exploratory and expressive digital media provide students with access to and potential for engagement with creative mathematical thinking (CMT) in unprecedented ways (Hoyles & Noss, 2003). Yet, new designs are needed to provide researchers and teachers with new ways of thinking about, teaching mathematics and supporting students' learning and engagement with creative mathematical thinking across all levels of education.

In this article, we present the collaborative work of digital educational media designers, for the design of a new kind of resources able to stimulate creative ways of mathematical thinking. Our study took place within the European Project called MC Squared (http://mc2-project.eu/). More specifically, within this project, we studied the collaborative design process of innovative digital resources, called c-book, aiming at developing the CMT of their end users. Innovative in the sense that c-books produced are the result of a creative process (Amabile, 1988; Černe et al., 2013), perceived as social creativity in the collective design action. This article has been driven by the purpose to highlight design choices for the promotion of CMT during the design of a c-book named "Math for Biology".

This article starts by discussing context, concepts and theoretical background emphasizing the concepts of c-book, creativity and CMT, refining our research questions in Section 2. Section 3 describes the methodology and data collection. Section 4 focuses on the design of "Math for Biology" c-book considering design choices and promotion of CMT components. It also provides choices analysis for CMT promotion. Section 5 reports on the a priori evaluation of CMT affordances. It ends with concluding remarks bringing forward factors influencing creativity during the collaborative design of digital resources in Section 6.

2. Context, Concepts, Theoretical Background, and Research Questions

The European MC Squared (MC2) project brought together four communities of designers of different nationalities (British, French, Greek and Spanish). The four communities gathered designers with various and different professional expertise, acting in a socio-technical environment under development. The goal of each of the designer communities was to produce digital mathematical resources, called c-books, aiming at developing the CMT of their users. The premise was: the higher the social creativity among the designers, the greater the potential of the c-books to develop the CMT of their users. Three cycles of c-books' production have been organized one after the other and the knowledge produced in each cycle was reinvested in the following one.

2.1 c-books

The c-books (Fig. 1) are one of the products of the collaboration amongst the designers with manifold professional expertise (e.g., mathematics, technical / technological, pedagogical and / or didactics).

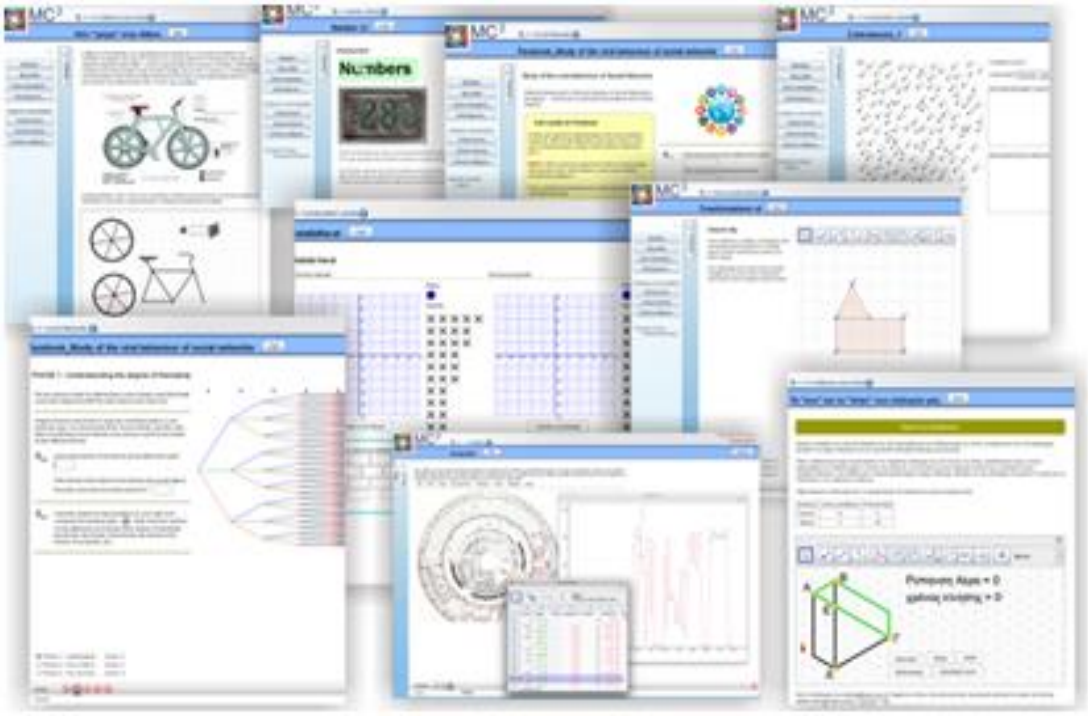

Fig. 1: Examples of c-books

These are mathematical digital books that contain text, pictures, videos and many interactive mathematical widgets including dynamic geometry software, dynamic algebraic systems, numerical calculation software allowing the use of multiple representations, to check some answers and/or to get feedback.

2.2 Community of Practice and Community of Interest

On one hand, a Community of Practice (CoP) (Wenger, 1998) is characterized by a common domain of knowledge and a common language with members who share a common repertory of resources. On the other hand, a Community of Interest (CoI) "brings together stakeholders from different CoPs to solve a particular (design) problem of common concern", (Fischer, 2001, p. 4). Its members have various areas of expertise and are only together for the time of solving the problem at stake, unlike a CoP. Furthermore, Fischer (2001) considers a CoP less creative than a CoI because a CoI provides unique opportunities to transcend individual perspectives. Therefore, the designers of the c-books who belong to different CoPs, have different domains of expertise and are together for designing collaboratively innovative resources are considered to constitute a CoI. Four CoIs (French, Greek, Spanish and British) have been involved in the creation of c-books (Essonnier, 2024).

2.3 Creativity and Creative Mathematical Thinking (CMT)

The c-books should enhance the creativity of their end users, in the *p-creativity* perspective, i.e. *psychological* creativity, or *little c* perspective (Boden, 1994; Craft, 2000). It concerns ideas that are considered creative at least in the mind of the person who proposed them. Therefore, it concerns not specifically gifted students but any students.

Regarding creativity, Csikszentmihalyi (1996) brought to the fore two opposite types of thinking: *divergent thinking* and *convergent thinking*. Divergent thinking generates novelty, new conclusions, and is based on four components: (1) **fluency** (the mental capacity to generate a large number of ideas); (2) **flexibility** (ability to move from one perspective to another), (3) **originality** (associations of unusual ideas), and (4) **elaboration** (the capacity to develop and extend to a high degree an idea). Convergent thinking involves (and allows) the evaluation of ideas and problem-solving processes by judging their relevance and usefulness. Moreover, according to Hennessey and Amabile (1999), the assessment of creativity can be carried out through creativity components and in a consensual manner based on the *middle c* perspective of creativity (Moran, 2010).

Based on these considerations, we have defined CMT situated in the *little-c* paradigm as follows. The CMT conceptualization is inspired by Guilford's (1950, 1967) and Torrance (1974) creative cognitive components (fluency, flexibility, originality, elaboration) to which we added social and affective aspects (Daskolia, 2016; Trgalová, 2016), applied to a mathematical problem or situation. **Fluency** means the individual's ability to pose or come up with many mathematical ideas or configurations in a short time. **Flexibility** refers to the individual's ability to vary the approach or suggest a variety of different

methods. **Originality** means the individual's ability to try novel or unique approaches. **Elaboration** is the individual's ability to redefine a single mathematical problem or situation to create others (El-Demerdash, 2010), (El-Demerdash, et al., 2019), (Essonnier, et al., 2021). **Social aspects** are stimulated by the user's interactions with other users, by developing students' mathematical communicative skills, or by providing users with opportunities for competing. Finally, **affective aspects** are based on the promotion of engagement by generating a perception of mathematics' usefulness, either in everyday life, or inside the mathematical context, or a feeling of pleasure, fun, challenge (narratives, games, etc.), or aesthetics.

2.4 Research Questions

Hence, we refine our research questions as follows.

RQ1: Which of the six components of CMT: fluency, flexibility, originality, elaboration and, social and affective aspects have been better integrated and promoted within the designed c-book by the designers?

RQ2: What is the priori potential of the designed c-book to develop CMT in their users?

RQ3: How have the design choices of the CoI members impacted the CMT affordances of the c-book?

3. Methodology and Data Collection

Alongside the production of c-books, knowledge was produced and shared within CoIs. To take them into account, our evaluation and study methodology was enriched after each cycle of production with this new knowledge. Therefore, the methodology has therefore evolved throughout the c-books three cycles of production and has been the more elaborated for the third cycle of the c-books produced.

Data of the design process were collected from the c-book final version, the official project reports (Mercat, 2014, 2015; Trgalová, 2016), meeting notes, part of the emails exchanged by the designers, the access to shared documents used for the design and the designed c-book

3.1 Methodology Highlighting Design Choices

From the information gathered into the c-book description in the appendix in Trgalová (2016, pp. 19-38) and by reviewing the c-book produced, it is possible to infer some design choices and to give an interpretation of the meaning of the CMT components made by the designers, showing how the latter had operationalized the CMT components during the design process of the c-book.

3.2 Evaluation of CMT

The methodology for evaluating CMT affordances of the c-book, is the one used in the MC2 project (Daskolia, 2016). To measure CMT and to make comparisons between the CoI(s), explicit and common criteria were needed. Common

features were decided and shared based on the literature review and many discussions among the four CoIs. So, a common grid based on a Likert agreement scale and shared by the four communities of interest has been elaborated for the third cycle of c-books production, called "CMT Affordances Grid" (See Appendix I) (Trgalová, 2016). This grid consists of three sections. The first 13 aim at evaluating the c-book affordances towards the development of creative mathematical thinking in the end users. These items address the c-book affordances such as nature of the activities or variety of representations of mathematical concepts at stake and ask the evaluators to what extent these affordances are likely to enhance the user's cognitive processes (fluency, flexibility, originality, elaboration). The second and third sections deal with social and affective aspects of the c-book that are likely to impact the users' intrinsic motivation and thus enhance their creative mathematical thinking. The grid enables us to assess the CMT affordances of each c-book produced with explicit and measurable criteria. It allows us to obtain a score for each component between 1 and 4. For this respect, the responders were asked to evaluate the items in relation to each one of the four cognitive components of mathematical creativity on a scale from 1 (weak affordance) up to 4 (strong affordance). There was an extra option called N/A in case the affordance was not applicable for the specific item. In detail, the statements of evaluation for the quantitative data could be described as follows:

- N/A means that the corresponding item is not present in the c-book;
- Value 1 means "no affordance", i.e., the corresponding item is present in the c-book but it does not foster the given dimension of the CMT;
- Value 2 means "weak affordance", i.e., the corresponding item is present in the c-book and might foster the given dimension of the CMT, but it is rather unlikely;
- Value 3 means "good affordance", i.e., the corresponding item is present in the c-book and might foster the given dimension of the CMT;
- Value 4 means "strong affordance:", i.e., the corresponding item is present in the c-book and is likely to foster the given dimension of the CMT

The evaluation of the CMT affordances of this c-book was done by three members of the French CoI, who were not involved in its design. It was organized in three steps. First, the evaluators had to use the c-book and be acquainted with the affordances. Second, the evaluators had a teleconference organized by the main designer of the c-book to address possible evaluators' needs for clarification or understanding. Third, the evaluators evaluated the c-book affordances based on the grid using an online form prepared for this purpose (See Appendix II).

4. Design Choices for the c-book "Math for Biology" and Promotion of CMT Components

This c-book[1] aimed at introducing the concept of biomathematics as a scientific terminology and its study fields such as biostatistics, mathematics of genetic engineering, etc. All the activities are designed around real-life situations to foster students' motivation and engagement. The c-book was designed to allow users to work in a non-linear manner according to their interests. It is structured in four sections. Section 1 is the introduction, section 2 is about mathematical approximation, section 3 about golden ratio and section 4 about spirals in nature. These sections are interrelated to each other by a narrative featuring a group of children visiting a Fauna and Flora Park, a context that ought to be familiar to users to engage them in the activities. This c-book is intended to be used with high school students (Grades 10-12).

Beforehand, it is important to point out that the designers are not biology majors, they are mainly mathematicians, so the c-book was developed from a mathematician's point of view. Biology was only the context chosen for mathematization. The designers' perspective was to model biological traits, physical ones, using mathematical tools to help users to think of mathematics as a modeling tool, to build meaning around mathematical concepts all the while developing their CMT skills, for supporting engagement of students as well. The students should choose a mathematical model after conducting some observations and/or calculations from real data.

First, we note that four design principles were at stake for the elaboration of this c-book. They are described in section 5.1 below. Second, we bring to the fore that the c-book is designed to enhance users' CMT through its components (fluency, flexibility, originality, elaboration, and affective and social aspects). The way the designers have interpreted these components through their design choices is detailed in section 5.2. Alongside developing CMT in users, another aim of the c-book "Maths for Biology" was to help students to build meaning in mathematics through modeling, but also to enable them to think of what means modeling, its limits, its power, and its usability. The designers have chosen mathematics modeling to help students on the one hand to make connections between mathematics and the real world, such as the biological world, and on the other hand to reflect on the assumptions leading to the choice of a mathematical model. We know that "*Mathematical modeling raises fundamental questions about the relationship between the real world and mathematics.*", as stressed by Greer, Verschaffel, and De Corte. (2002, p. 273) who also were quoting Freudenthal (1991, p. 32, in Greer & al., 2002, p. 273):

[1] Available online at: http://mc2-project.eu/

> "*Mathematics has always been applied in nature and society, but for a long time it was too tightly entangled with its applications for it to stimulate thinking on the way it is applied and the reason why this works ... money changers, merchants and ointment mixtures behaved as if proportionality were a self-evident feature of nature and society ...*
> *... Modeling is a modern feature. Until modern times the application of rigorous mathematics to fuzzy nature and environment boiled down to consciously ignoring all of what had appeared to be inessential perturbations spoiling the ideal case.*"

Thus, depending on the mathematical model chosen, solutions to problems can be very different.

4.1 Four Design Principles at Stake

During the third cycle of c-book productions within the French CoI, four design principles, described in the following sections, emerged.

1. Personalized Non-Linear Path and Narrative Affordances

First, the c-book consists of four sections interrelated through a narration about children visiting a Fauna and Flora Park, in accordance with Laurillard et al (2000, p.2) point of view about narrative and non-linear path:

> "*Narrative provides a macro-structure, which creates global coherence, contributes to local coherence and aids recall through its network of causal links and signposting. [...] Multimedia has a non-linear format, which cedes control over both sequence and internal relationships to the users. They decide on the order of the material, and they determine the nature of the link between one section and the next. In this context, the narrative line cannot be held entirely by the program. Instead, it results from an interactive collaboration between the user and the program. The learning activities linked to narrative in a linear medium are likely to be different, therefore, from those in a non-linear medium such as multimedia".*

The narrative is based on:

> "a group of children visits the famous F2 Park (Fauna and Flora) with their biology teacher. They go through various greenhouses, pathways, and enclosed plots in which they explore the amazing diversity of life on earth: from dinosaurs to massive mammals, from fossilized ferns to giant trees.".

Section 1 introduces the meaning of the term biomathematics, its fields of study, and ends up giving users the opportunity to pick and choose among different activities bound to biomathematics. The biomathematics context has been chosen for stimulating the users' curiosity, motivation, for its familiarity to them and for favoring mental manipulations. At the end of the introduction, page 3, the storytelling starts, users are free to choose their path of learning thanks to

embedded hyperlinks. The latter allows users to autonomously navigate according to their interest in the other sections. Section 2: demonstrates the need of mathematical approximation as a tool to model biological traits to estimate magnitudes (length, thickness, weight), and it enables users to reflect on magnitudes. In section 3, golden ratio is a pretext search for doing mathematics by looking at its occurrences in nature, paintings, and architecture. The c-book ends with section 4, where users are exposed to different kinds of spirals in nature. Users can upload their spiral pictures and try out some formulas that fit them, using different digital widgets based on algorithms, algebra and Fibonacci numbers. Finally, they can build their own spiral-based artwork to stimulate new idea associations and originality. The activities in the c-book engage the users in compelling problems, and the widgets set up orient, facilitate mathematical exploration and/or allow creative ways to think about them, investigate and solve them. Thus, the storytelling approach is supported by interactive widgets and promotes the affective aspect component of CMT to encourage students' own exploration for meaningful learning. So, the designers have used a story, as a guideline, to enable users to choose their own path throughout the c-book activities and to keep them focused while they go through it according to their interest, their curiosity, their mathematical and technological prior knowledge, such as: digital tools, ratio and proportion, function, curve, usual solids, usual volume formulas. We put forward that the story was a means for the designers to encourage users to be active to stimulate their engagement, all the while guaranteeing the unity of the c-book activities.

2. Technology Added Value to Promote CMT

The designers were eager to support the development of the CMT components of users with relevant specific widgets and feedback. They seek to develop the socio-technical environment to favor the CMT affordances. We present three examples of designers' use of the socio-technical environment which highlight the technological added value provided by the development of this environment.

(1) The first example is the cross-widget communication between Cinderella and EpsilonWriter, two mathematical educational software of the environment. The idea behind cross-widget communication is to send measurements done within Cinderella to EpsilonWriter where users can write text and mathematical content, as well as calculations. The purpose of the designers was to facilitate users' work, to lead them to compare the ratio values to enable them to make a conjecture, i.e. whether the golden ratio models the dimensions of a bee or not (Fig. 2). The designers aim at fostering flexibility in the users, for instance about the way they look at a bee, i.e. to see it through its measurements for a mathematically modeling purpose.

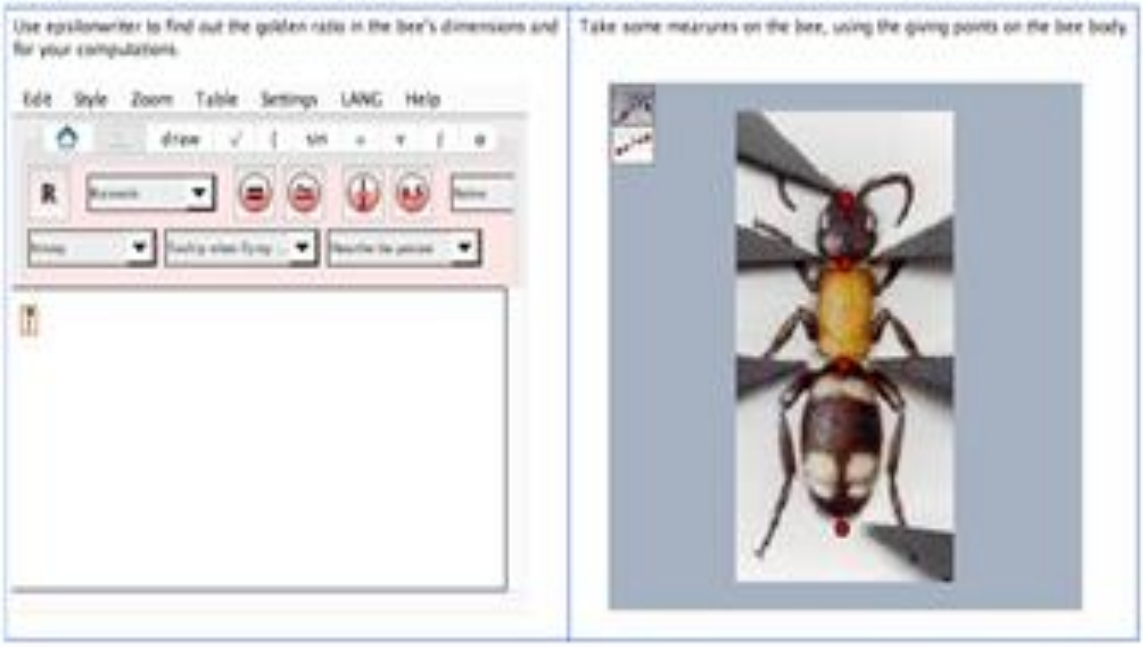

Fig. 2: Cross-widget communication to allow sending information between Cinderella and EpsilonWriter

(2) The second example shows a digital tool built on mathematical properties to explore the real world looking for these mathematical properties. The digital golden ratio compass tool (Fig. 3) is a DGS widget enabling users to discover proportions seemingly like golden ratios in other contexts such as in insects, and in plants, artwork, etc. Users are encouraged to make trials on many things to boost fluency. They should look for a picture, upload it and check with the digital compass whether its dimensions satisfy the golden ratio or not. They can share their thoughts about this compass and their findings via EpsilonChat and EpsilonWriter, two of the Epsilon tools. One of the designers' purposes is to obtain as many samples as possible, to make users reflect on proportions and ratio bound to the golden ratio. The main added value here is meshing three complementary widgets in the same educational environment. Nevertheless, we can raise an issue. As the users are working on pictures or photos there might be some distortions on lengths' proportion. This is a mathematical question that can emerge during this activity. Maybe the teacher ought to take the opportunity to raise the subject through EpsilonChat if the users do not think about it.

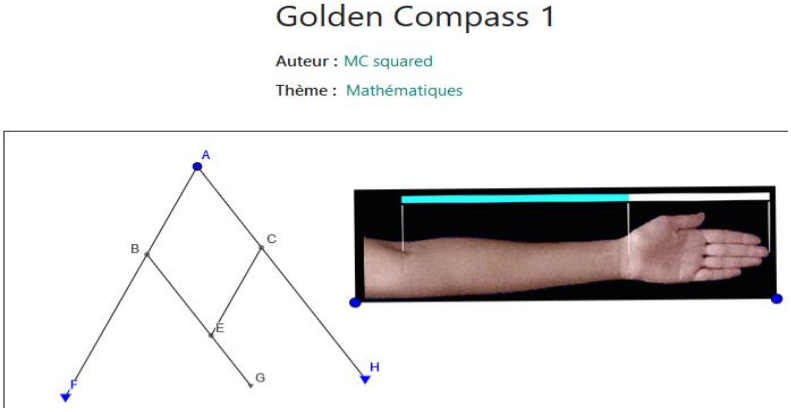

Fig. 3: Shows digital golden ratio compass GeoGebra tool

(3) The third example is about the socio-technical environment flexibility itself. The socio-technical environment allows the use of external widgets, software that couldn't be provided by the environment but are reachable by external hyperlinks. For instance, a 3D-GeoGebra widget that enables users to change the dimensions of a cone and a cylinder to get the same. It enables users' mental comparison and manipulation of solids for mathematically modeling living beings. This widget should help users to think of simpler shapes to model real world living beings to compute volumes and make comparisons amongst volumes of different living beings. It fosters an approximative approach and highlights that the real world should be sometimes simplified to apply mathematics formulas and reasonings. The interactive GeoGebra widget is supposed to help users to make their conjectures about volumes of two solids, i.e. the cylinder and the cone. The goal is to make users understand that the choice of magnitude is important to make sense for comparing two creatures.

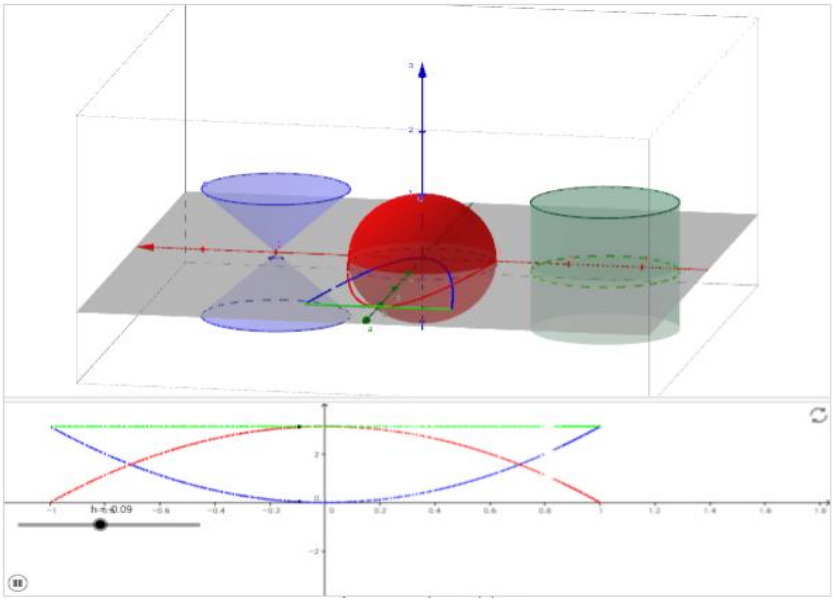

Fig. 4: Shows a GeoGebra widget of cylinder and cone
See (https://tube.geogebra.org/m/2552113)

Section 4, page 5, the designers provide users with another GeoGebra widget, embedding relevant construction tools (Polygon tool, arc tool), to help users to construct a Fibonacci spiral.

Technology has been a means for designers to lead users towards mathematization. This use of technology fosters flexibility to facilitate and to support reasoning in users.

3. Socio-Constructivism Approach - a Cultural Approach to Learning in French Didactics Tradition

The activities in sections 2, 3 and 4 have been developed based on the French socio-constructivist learning approach (Blum, et al., 2019) and the use of socio-constructivist principles to establish internal and explicit connections among mathematics, biology, and technology (Toma, et al., 2024). The designers gave preference to tasks that allow users to be active, to create their own experiences and knowledge, to make connections with prior knowledge, by offering opportunities for conjecturing, exploring, explaining, and communicating mathematically. The French CoI socio-constructivism tradition background in mathematics education aligns with their CMT representations emphasize social aspects of c-book users by integrating game-based widgets and mathematics communication digital tools to foster motivation. In this concern, each section ends with a meta-cognitive activity (detailed in the coming section) asking for reflection and collaboration to enhance social aspects through mathematics communications. Finally, a chat tool, called EpsilonChat, was implemented in many pages of the c-book for allowing social interactions amongst users.

So, the socio constructivism tradition in mathematics education of French CoI has specifically fostered social aspects of CMT. For this purpose, the designers enable social interactions through the implementation of EpsilonChat.

4. Meta-Cognition - Learning by Reflection on One's Own Work

To enable them to assess themselves the mathematical model chosen, its appropriateness.

Sections 2,3 and 4 of the c-book are ended by a meta-cognitive activity which has been designed to encourage the users to reflect in groups about their own mathematical activities and their learning by writing a short description of their work. The designers, through this task, try to stimulate users to make them active learners and to enable them to further understand, analyze and assess their own cognitive processes, such as reflecting on the appropriateness of tasks they have performed into the c-book and the mathematical model chosen to study the biological traits. This activity also allows users to develop their written mathematical communication skills through EpsilonWriter and EpsilonChat which are tools for communicating mathematically. This activity is also a way

to enhance the social aspects of CMT. It is called: "Students' collaborative learning documentation" (Fig. 5). Nevertheless, there is a limitation because it is not possible for the users to control their work, their answers from the feedback of the environment.

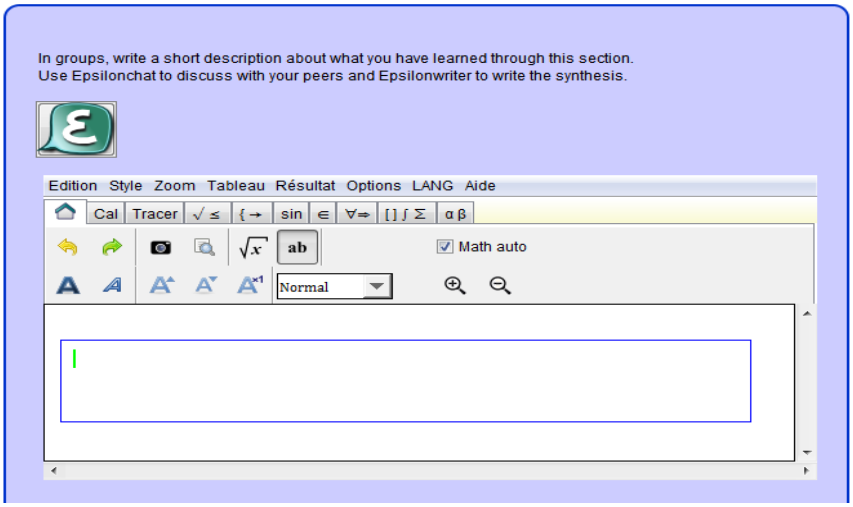

Fig. 5: Snapshot of section 3, last page, on the meta-cognitive activity

We observe that the design principles retained by the designers were bound to four CMT components, fluency, flexibility, affective and social aspects, and to their cultural background, socio-constructivism. In the following section, we are interested in bringing to the fore some means used by the designers to foster CMT in users.

4.2 Choices Analysis for the CMT Promotion

1. Affective Aspects

In the first section, "Introduction", page 1, a game, a Tquiz puzzle (Fig. 6), is proposed to check users' understanding of what living and nonliving organisms' concepts mean in biology. This game is supposed to support the affective aspects of CMT among c-book end users.

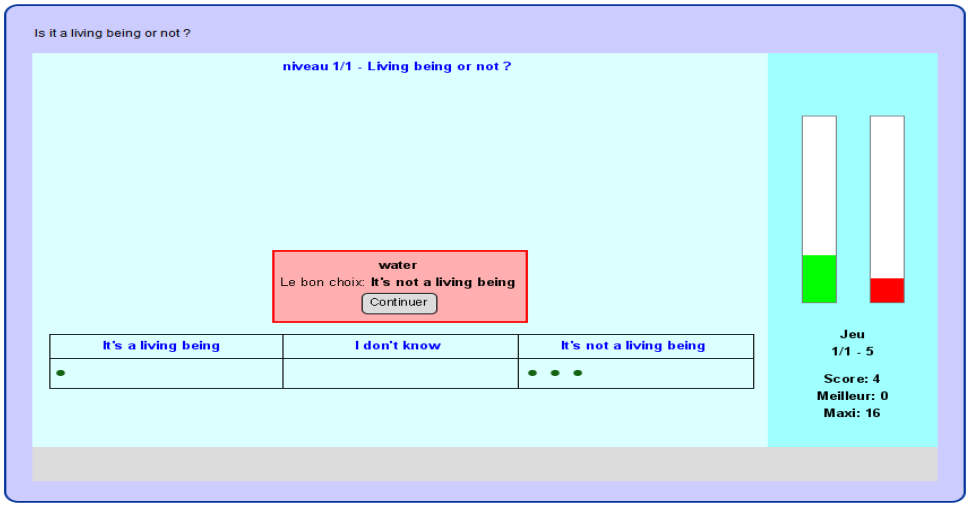

Fig. 6: Snapshot of the Tquiz puzzle on living and nonliving organisms

As noted in section 5.1, the storytelling also allows to enhance the development of CMT affective aspects. Therefore, section 2, page 1, we observe that the storyline started in the introduction continues: “An aquarium featuring a great variety of marine creatures” is a part of the park. Users are attracted by the most impressive exhibited species, amongst them is the blue whale, the largest animal ever lived on Earth. The children then enter a pathway by going to page 2 in section 2. In this page, the storyline continues: The “Giant Tree” pathway is bordered with enormous trees among which one catches the eye of all visitors: the “General Sherman” tree. Users can learn a lot about the blue whale and this giant tree from the info panels provided by the designers (Fig. 7)”. The colored panels created by the designers embed photos, maps, texts, and clickable links for motivating users’ active behavior.

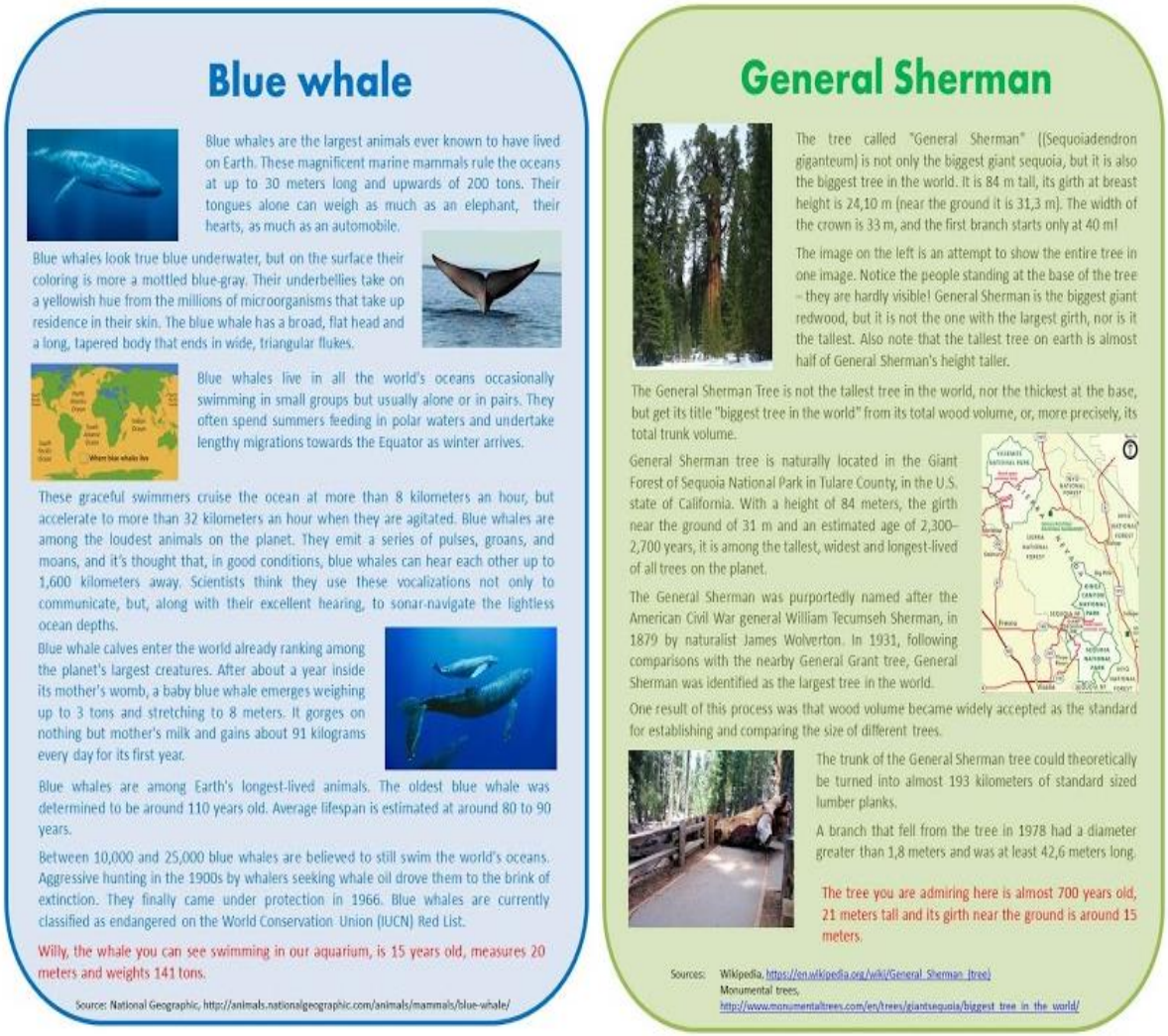

Fig. 7: Panels with information about the “blue whale” and “General Sherman”

In page 3, the designers used an argument between students to orient their reflection toward the two creatures and their comparison by various magnitudes. Here are the students' argumentations in the follow up of the storyline:

> "Impressed by the considerable dimensions of these two creatures", the children start comparing them:
>
> Jack says: "The tree is bigger than a whale because it is taller than the whale."
>
> Amy says: "I don't think so. The whale is bigger because it is thicker than the trunk of the tree even near the ground."
>
> Sean says: "Yes, I agree with Amy, the whale is bigger, but because it doesn't float while the whale does, it is heavier."
>
> Afterwards, we guide c-book users to decide on students' claims by the question: "What do you think about the students' claims? Which one is bigger: the whale or the tree?
>
> If you wish to know more about the whale, you can go to page 4, and if the giant tree is intriguing you, go to page 5 to learn more about it."

Pages 4 and 5 start with a text followed by a photo, and the possibility to watch short videos by clicking on a link, to attract students' attention and interests. The first short video (1:01 min) shows the biggest mammal on the planet earth: The blue whale and the second short video (3:05 min) takes users to California where the biggest tree on earth is: The "General Sherman". From the storytelling and the impressive dimensions of the two very big creatures, we infer that the designers aimed at motivating users to compare their magnitude and to feel the need to use mathematical modeling to be able to make comparisons.

Section 3, page 1, the designers have chosen to keep on the story. In the latter, students move to another area of the park called "temporary exposition" that exhibits the evolution of the human being. Students are particularly **interested** in the reconstituted fossil of Lucy (Fig. 8).

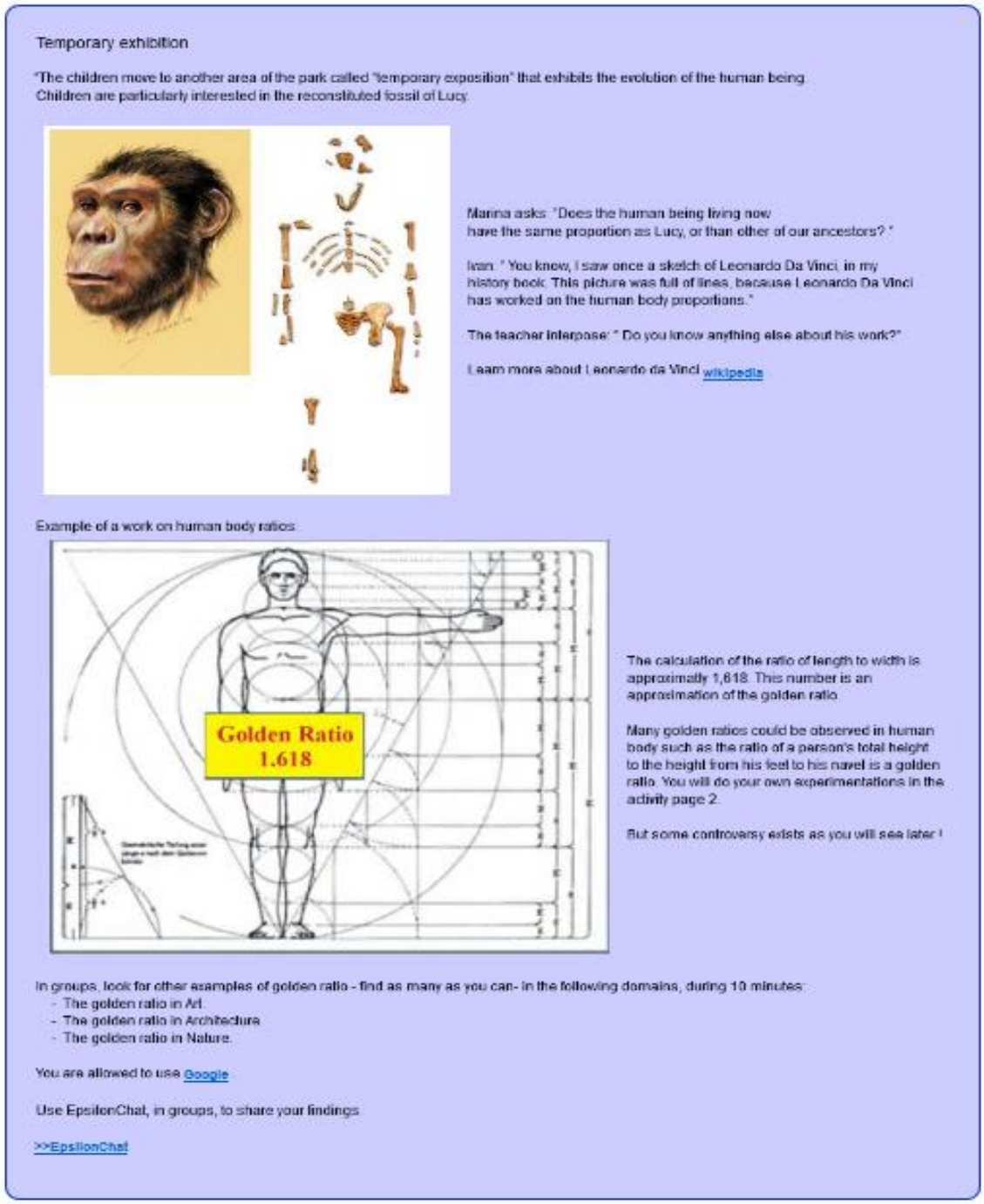

Fig. 8: Snapshot of section 3, page 1

Here is the story:

"Marina asks:" Does the human being living now have the same proportion as Lucy, or than other of our ancestors?"

Ivan:" You know, I once saw a sketch of Leonardo Da Vinci, in my history book. This picture was full of lines, because Leonardo Da Vinci has worked on the human body proportions."

The teacher interposed:" Do you know anything else about his work?""

The designers also provide users with a hyperlink to learn more about Leonardo Da Vinci artwork. Users could discover that the proportion highlighted by Leonardo Da Vinci artwork is the Golden ratio. The designers put another hyperlink to facilitate and to orient users' research, and to enable them to be more active by choosing whether to use hyperlinks or not. It's a way to leave them some small responsibilities.

Section 4, page 1, we can observe that the story continues with a visit to the museum of the park. The students of the story are in front of the fossil photo (Fig. 9) from the London Museum. Their teacher explains that we can observe many spirals in nature. These spirals have always inspired humans, because we can find many sketches, sculptures, and pieces of artwork using this shape.

Fig. 9: Shows a fossil picture from London Museum

The designers seem to use the story to keep some coherence in the c-book, and it enabled them to introduce the activity and motivate users. In addition, page 2, the designers gathered three photos of artworks (Gustav Klimt "L'arbre de vie", a Triskell and celtic art) with different kinds of spirals. The photos are used for putting forward the aesthetic aspect of spirals highlighted by the artworks, to incite interest towards spirals and enhance curiosity of users about these spirals. This appears to be a means for the designers to lead the users to question how the spirals were drawn, the relationship between these spirals and mathematical modeling such as curves or functions to favor connections and generalizations.

Therefore, we note that games, storytelling, maps, hyperlinks, photos and short videos in relation to the narrative seem to have been a way for the designers to

trigger affective aspects of the users. They also relied on aesthetic features of artwork, fossils, and nature.

2. Social Aspects

The designers embedded the EpsilonChat widget to mainly enable interactions between users, as already stressed in section 4.1. More specifically, the concept of "biomathematics" as a scientific term and its study fields are introduced on page 2 of the introduction. In this page, students are asked to look for many different and varied examples of situations that show the integration between mathematics and biology using Internet resources to build their knowledge regarding the study fields of biomathematics and discuss these examples with their peers through EpsilonChat. In section 3, users ought to investigate the golden ratio on the Internet and share their findings using EpsilonChat. Through all sections the designers explicitly asked users to discuss their observations, conjectures, findings through the chat tool.

Section 2, page 3 proposes a practical activity where users can experience a calculation of the golden ratio using the GeoGebra spreadsheet widget to find out one of the golden ratios of human body dimensions. For this purpose, the designers created tasks where users have to collaborate to gather real data from their own bodies and do computation with these data (Fig. 10). They favored a context close to the users to stimulate curiosity of the latter with the support of digital technologies.

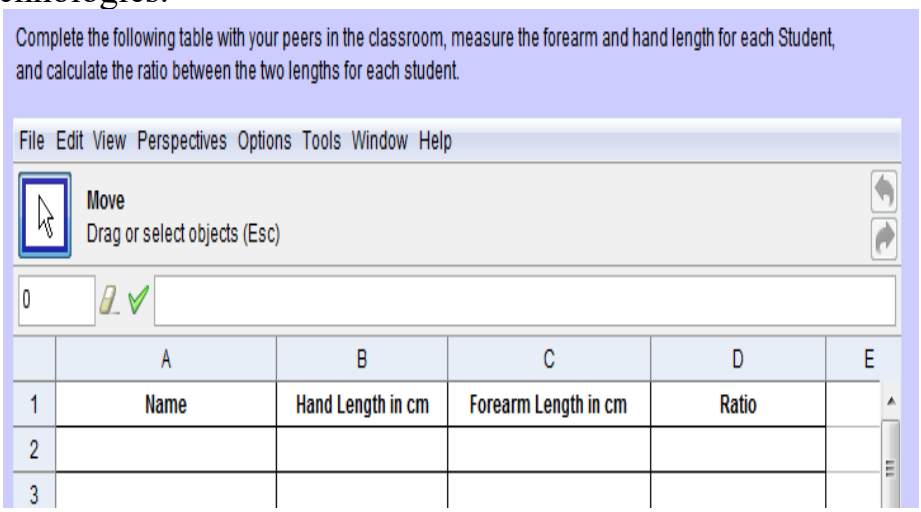

Fig. 10: Shows a GeoGebra spreadsheet to be used in the practical activity calculating golden ratios in the human body

So, the chat tool has been a means for asking users to share either their findings or conjectures, or to collaborate for data collection. Hence, the chat tool and the collaboration were employed by the designers for allowing interactions and fostering social aspects within a digital environment.

3. Fluency

Throughout the c-book, there are many occurrences of activities to foster fluency, as follows. In the golden ratio section 3, on page 1, users were asked to collectively use EpsilonChat, and to look for **many** examples of golden ratio related to art, architecture and nature in 10 minutes. In the spiral section, on page 2, users were asked to use the Internet to find **more examples** of spirals. In the same section, on page 4, users were also asked to list **as many** Fibonacci numbers **as** they can in 2 minutes using the EpsilonWriter mathematics communication tool. So, for the development of fluency, the designers used the words and expressions such as: **"many"**, **"more examples"**, **"as many as"**.

4. Elaboration

Section 2, last page, the designers proposed a problem posing activity to foster the elaboration of the users based on other examples such as a vow, a crocodile. This was the only task encouraging elaboration into the c-book. Furthermore, in this c-book flexibility and originality components combined with another component, such as detailed in the following section.

5. Interrelated Components

Fluency and Flexibility

Many tasks of the c-book combine the development of fluency and flexibility together. From the very beginning of the c-book, in the introductory section, page 2, users are invited through the EpsilonChat tool (Fig. 11) to come up with many varied examples or situations that show the integration between mathematics and biology. Likewise, in the spiral section, users are asked to come up with as many and varied spirals as they can. Both above examples were designed intentionally to develop both components together. In detail, the designers ask users to use the Internet to look for **more different and varied examples** of artworks which are based on the use of spirals. Section 4, on page 3, proposes **many different and varied examples** of spirals in nature which inspire artists and mathematicians to look for models and formulas that make use of these spirals. Users can play with the provided GeoGebra widget to model these spirals and look for the nearest formulas to construct them. Evenly, section 4, page 4, users are asked to list **as many** Fibonacci numbers **as** they can and write them down in a provided EpsilonWriter worksheet in 5 minutes after being introduced to these numbers by question followed by a short video (6:24 min) presenting the Fibonacci numbers in an elegant way and shows one geometric representation of these numbers.

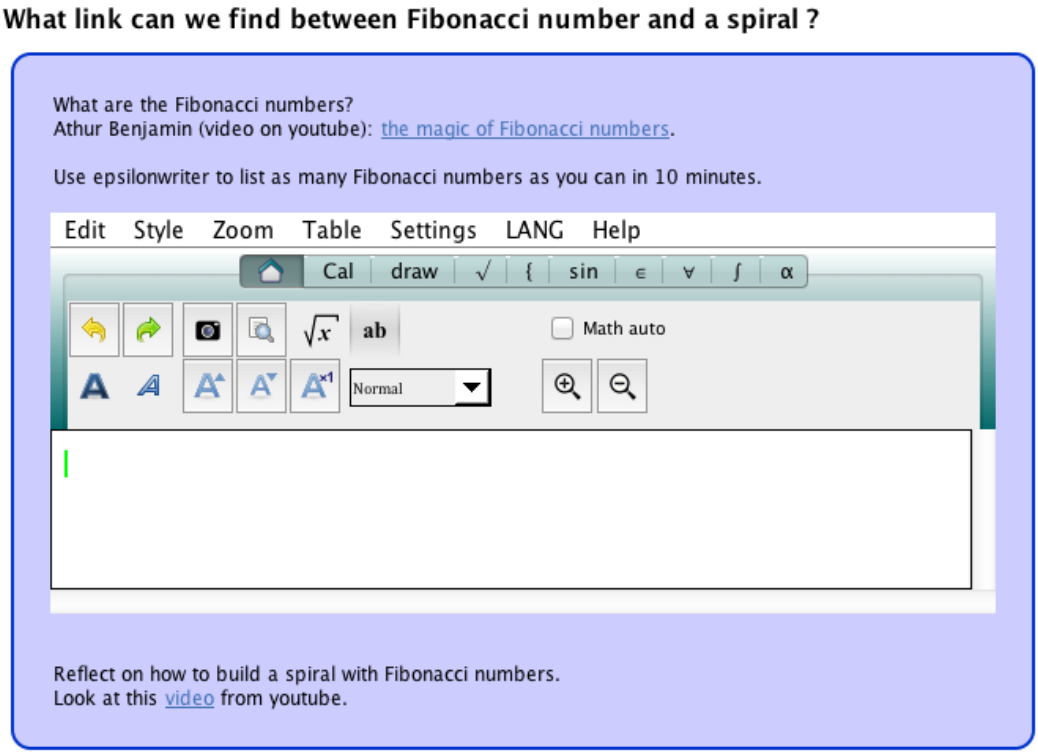

Fig. 11: Shows an EpslionWriter widget to reflect on Fibonacci numbers

We saw that the designers formulate tasks using the words **"different"** or **"varied"** to enhance **flexibility**, with the words **"many"**, or **"as many"** as to refer to **fluency**, adding in between the preposition **"and"** to stimulate both components in the users while performing these tasks.

Fluency and elaboration

The users were provided with three examples of pictures (an aloe polyphylla, the half of a red cabbage and a snail) for developing fluency, and simple algebraic formulas which "don't quite fit" and need to be adjusted, hence the formulas need to be elaborated to fit better. In this case, CMT is fostered mainly in the direction of fluency and elaboration and depends on the personalized feedback provided by the environment (here visualized feedback coming from the graph of the edited formula).

Flexibility and Originality

As a means used by the designer to enhance creativity can be seen in section 4, page 7, 8 and 9. Page 7, titled: "Other types of spirals", we see a spiral realized with a widget (Fig. 12) designed by the Greek community. The spiral was drawn with an algorithm in logo programming language. The users are asked to modify the algorithm and watch what happens then (visualized feedback from the environment).

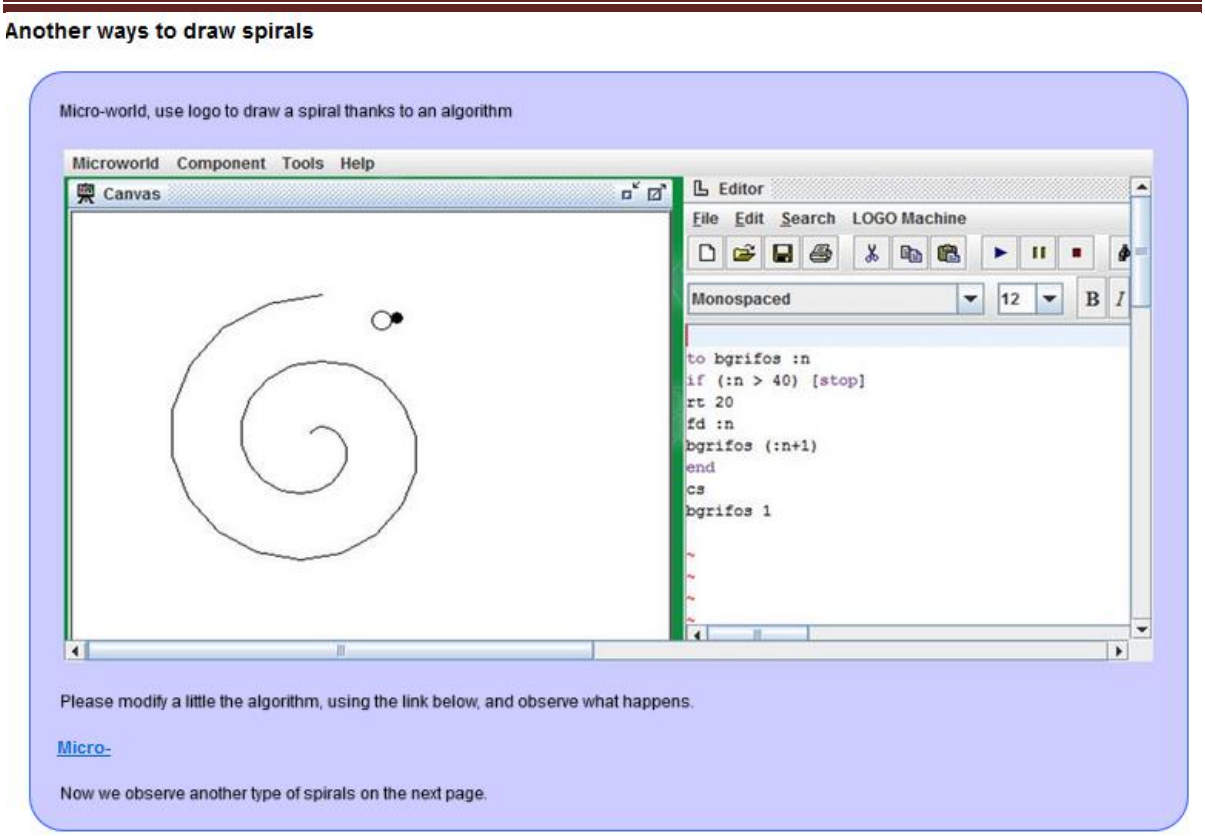

Fig. 12: Using logo programming to draw a spiral

Page 8, titled: "Modeling spirals with squares", the designers ask users to observe another new type of spiral. It shows points with integer coordinates placed on a plane, designed with Cinderella - dynamic geometry software. Users are invited to write their conjecture about the mathematics behind it. Finally, page 9 titled: "artwork", the designers ask users to create their own artwork based on what they have learnt about spirals in the previous tasks of this section. So, the designers through many examples gave the opportunity to the users to build a broad knowledge about spirals favoring flexibility. Then, by using the word "own", we deem that they try to foster users' originality. So, the designers used flexibility to propose several perspectives and the word "own" as a lever for originality.

So, the purpose of some of the designed tasks is not only to promote one component but also another one.

5. CMT Affordances A Priori Evaluation Analyses

The chart, shown in Fig. 13 below, represents the evaluation of the cognitive components of CMT from the evaluators' point of view, considered as experts. The height of the bars represents the distribution of the four components for each item (fluency, flexibility, originality and elaboration), while the thickness represents the meaning between the four aspects for each question.

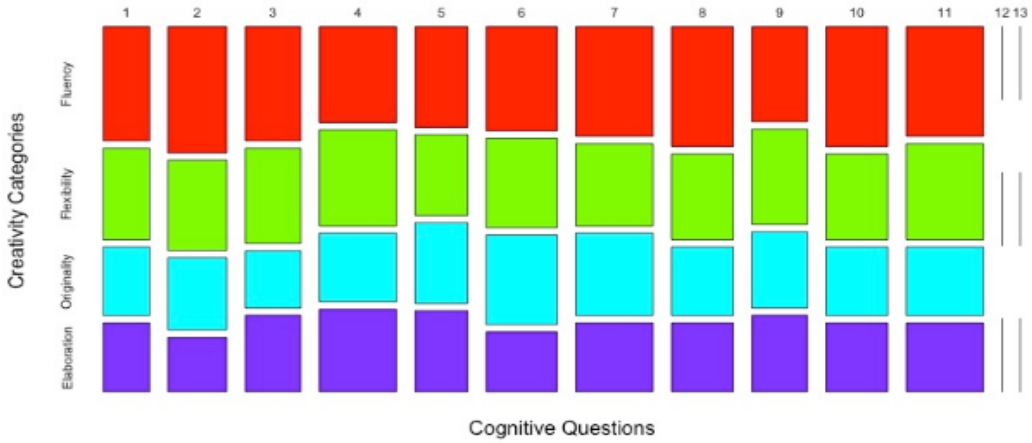

Fig. 13: Evaluation of CMT cognitive components from the evaluator experts' point of view

On the one hand, only the items 12 and 13 were assigned by the evaluators the N/A value (no affordance) for the four cognitive components. It means that the designers did not include any foreseen creative constructs in the c-book. That is, this c-book does not include non-standard problems calling for mathematical solutions (item 12) and does not include half-baked constructs (more specific to the culture of the Greek CoI) that call for intervention (item 13). On the other hand, the evaluators considered that the c-book provides users with opportunities to establish connections among different knowledge areas and mathematics includes interdisciplinary, cross-disciplinary, and external connections (item 4), stimulates or encourages exploratory activity and user experimentations (item 7) and stimulates or encourages generalizing mathematical phenomena, going from concrete cases to general ones or generalizing real world phenomena using mathematics (item 11).

The results of the a priori evaluation of CMT affordances of the c-book are summarized in Table 1 below. The quantitative data for each component computing the mean from "No Affordance" (scored 1) to "Strong Affordance" (scored 4), from the scale defined to evaluate the c-book.

Table 1. CMT Evaluation Summary

Parameters	Fluency	Flexibility	Originality	Elaboration	Social	Affective
Mean	3.03	2.46	2.11	1.92	2.83	3.33

Except the elaboration component, all other components exist in the range of "weak" to "good" affordances. The elaboration got a value of 1.92 which means that this value is quite close to 2, i.e. "weak affordances". The highest value for this c-book in terms of cognitive aspects was fluency for which the value achieved the rank of "good affordance". It means that, in general, the c-book is judged to have potential to boost the students' development of their ability to

provide many responses or situations by changing one or more aspects of the original situation. In section 4, we saw that the designers created many tasks to develop fluency in the users but proposed very few tasks to favor elaboration. It has been difficult for the designers to elaborate tasks by fostering elaboration for this c-book. The tasks created for elaboration use relevant feedback from the digital environment and problem posing principles. Flexibility and originality are the cognitive components in between the two others, meaning that the evaluators perceive a slight potential to foster students' ability to come up with diverse strategies to solve a mathematical problem or challenge (flexibility) or to provide novel responses (originality). Concerning social and affective aspects, this c-book has good affordances to enhance affective aspects and that could be traced back to the nature of the c-book content as it deals with biological context which seems to be good in enhancing and promoting it. The score for social aspects is a little lower and bound to the chat tool as shown in section 4 above. Table 2, below, collects the 13 items of the questionnaire to show correlations among the four cognitive components of CMT.

Table 2. Correlation values of CMT components

	Fluency	**Flexibility**	**Originality**	**Elaboration**
Fluency	1.00	0.89	0.67	0.75
Flexibility	0.89	1.00	0.69	0.90
Originality	0.67	0.69	1.00	0.60
Elaboration	0.75	0.90	0.60	1.00

We can notice that the correlations range from moderate (considering a significant value of $r > 0.6$ ($p = 0.05$)) - showing the relationship between originality and elaboration to strong (considering a significant value of $r > 0.89$ ($p = 0.05$)) shows the relationship among the three other components each two together (fluency, flexibility, and originality). Considering these correlation values, we may conclude that fluency, flexibility, originality and elaboration can be fostered at the same time. In line with findings reported in the literature (Mann, 2006) and (Haylock, 1997) correlations among the four cognitive -each two together- components of CMT are strong between some cognitive aspects. It means that, considering the significant value of correlation coefficients, we may conclude that the four cognitive components of CMT fluency, flexibility and elaboration can be fostered at the same time. For example: In the section entitled "Spirals in Nature", users were asked to browse the Internet and look for as many varied and different examples of artworks that are based on the use of

spirals. This example is intentionally designed to promote the development of the "Fluency" component through asking users for many examples. At the same time, it also literally promotes the "Flexibility" component by asking users that these examples should be different and varied. In addition to the development of fluency and flexibility, originality and elaboration could have been also developed while the users try to figure out the formula for curves of these spirals and interact via the chat tool to elaborate on that. In section 4 above, we also brought to the fore that tasks created to foster originality are intended to develop flexibility as well, in accordance with the score of correlation between originality and flexibility (0,69).

The evaluators did not comment that much on their quantitative evaluation. However, they raised the need for good orchestration to achieve the goals intended by the c-book. They seemed to perceive the CMT affordances the designers were aiming at because some significant results may be traced back by the nature of the tasks in the designed activities. The c-book was well evaluated in terms of fluency and affective aspects; these components seemed to be easier to promote for the designers. Even though the c-book activities are designed to call for students' fluency (they are invited to come up with as many situations as possible related to the original one), flexibility, originality and elaboration are fostered by providing the students with a rich environment in which they can come up with varied and novel situations.

6. Conclusion

To answer **RQ1**: Which of the six cognitive components of CMT: fluency, flexibility, originality, elaboration, and social and affective aspects have been better integrated and promoted within the designed c-book by the designers?

First, we have highlighted four design principles that were at stake during the design process bound to cultural context and to technology: a nonlinear path, a socio constructivist approach, added value of technology and meta-cognition of users. Then, these principles enabled the designers to develop mainly fluency, effective and social aspects of CMT, and flexibility in mathematization. While proposing tasks to encourage mathematical flexibility and originality, the designers used the word "own" in some following tasks to stimulate originality of users. We have also noted that the way of formulating some tasks favors fluency, flexibility and originality. So, the formulation has been used by the designers to promote some components. Nevertheless, tasks to foster elaboration and originality in the users seem to have been difficult to create for the designers in this digital environment, especially relevant and personalized feedback depending on the actions of the users. Only one task, based on problem posing, allowing elaboration has been designed. As problem posing seems to be a good

way to foster for instance elaboration, we raise the question of why the designers did not implement more problem posing activities within this c-book.

To answer **RQ2:** What is the priori potential of the designed c-book to develop CMT in their users?

From the priori evaluation, this c-book is supposed to well develop fluency and affective aspects, social aspects depending mainly on the orchestration by the teacher, to weakly develop flexibility, originality and to not really develop elaboration.

To answer **RQ3:** How have the design choices impacted the CMT affordances of the c-book?

Indeed, the results on the CMT affordances reflect the design principles and choices of the members of the CoI who designed this c-book. Hence the design principles and choices have impacted on the potential to develop CMT in c-book users. This impact has been more positive for the affective aspect and fluency than for originality or elaboration. It would be interesting to reflect on design choices, principles enabling designers to develop all the CMT components, especially the weakest ones, within a digital environment, to enhance the CMT affordances.

Moreover, the CMT affordances evaluation was limited to **a priori evaluation** by field by CoI members considered as experts. Whereas it could have been beneficial if CMT affordances have also been evaluated from teachers and students' point of view to compare with the priori evaluation done by experts. This might be evoked in further studies. Last and not least, the teacher's role for the implementation of these resources should be very important in organizing and orchestrating the c-book utilization for achieving the goals envisioned by the designers about CMT.

Finally, there was one issue that we did not raise in this paper but that it is important to consider when designing technology. It is the sustainability of technology. Indeed, when investing in creating digital resources, these resources should be available and operational during a certain amount of time.

References

Akkerman, S. F. & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132-169. https://doi.org/10.3102/0034654311404435

Blum, W., Artigue, M., Mariotti, M.A., Sträßer, R. & Van den Heuvel-Panhuizen, M. (2019). European didactic traditions in mathematics: Introduction and overview. In: W. Blum, M. Artigue, M. Mariotti, R. Sträßer & M. Van den Heuvel-Panhuizen, (eds) *European Traditions in Didactics of Mathematics*. ICME-13 Monographs. Springer, Cham. https://doi.org/10.1007/978-3-030-05514-1_1

Boden, M. A. (Ed.). (1994). *Dimensions of creativity*. Cambridge, MA: MIT Press. https://doi.org/10.7551/mitpress/2437.001.0001

Csikszentmihalyi, M. (1996). *Creativity: Flow and the psychology of discovery and invention*. New York: HarperCollins. https://doi.org/10.1177/001698629704100309

Černe, M., Jaklič, M. & Škerlavaj, M. (2013), "Authentic leadership, creativity, and innovation: a multilevel perspective", *Leadership*, 9(1), 63-85, https://doi.org/10.1177/1742715012455130

Craft, A. (2000). *Creativity across the primary curriculum: Framing and Developing Practice*. London: Routledge / Falmer.

Daskolia, M. (2015). D2.2: *Operational definitions and criteria for measuring social creativity in the design of digital educational resources for CMT*. Deliverable of the MC Squared project. FP7, ICT-2013.8.1. Accessed 7 April 2025 from: http://mc2-project.eu/index.php/dissemination

Daskolia, M. (Dir.) (2016). D2.4: *A refined theoretical framework on social creativity in the design of digital educational resources for CMT*. Public report, Mathematical Creativity Squared, FP7 – ICT, Grant Agreement no: 610467, Accessed 7 April 2025 from: http://mc2-project.eu/index.php/dissemination

EC (2006). *Recommendation 2006/962/EC of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning*. Official Journal of the European Union, L 394, 10–18.

El-Demerdash, M. (2010). The effectiveness of an enrichment program using dynamic geometry software in developing mathematically gifted students' geometric creativity in high schools. Doctoral dissertation, University of Education Schwäbisch Gmünd, Germany, http://rgdoi.net/10.13140/2.1.1733.0247

El-Demerdash M., Trgalová J., Labs O. & Mercat C. (2019) Design and evaluation of digital resources for the development of creative mathematical thinking: A case of teaching the concept of locus. In: G. Aldon & J. Trgalová (eds) *Technology in mathematics teaching. mathematics education in the digital era*, vol 13. Springer, Cham. https://doi.org/10.1007/978-3-030-19741-4_7

Engeström, Y. (1987). Learning by expanding: An activity. *Theoretical Approach to Developmental Research. –Helsinki: Orienta–Konsultit*.

Engeström, Y., Engeström, R. & Kärkkäinen, M. (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities. *Learning and instruction*. https://doi.org/10.1016/0959-4752(95)00021-6

Essonnier, N., Barquero, B., Papadopoulos, I., Trgalová, J. & Kynigos, C. (2024). Communities of Interest as a Context for Creativity in the Process of Designing Digital Media for Mathematical Learning. In: B. Pepin, G. Gueudet, & J.

Choppin (eds) *Handbook of Digital Resources in Mathematics Education. Springer International Handbooks of Education*. Springer, Cham. https://doi.org/10.1007/978-3-031-45667-1_40

Essonnier, N., El-Demerdash, M. & Trgalová, J. (2021). Comparing social creativity among designers with creativity of mathematical digital resources produced. *The 14th International Congress on Mathematical Education*, Jul 2021, Shanghai, China. https://hal.science/hal-05161600v1/document

Fischer, G. (2001). Communities of Interest: Learning through the Interaction of Multiple Knowledge Systems. In S. Bjornestad, R. Moe, A. Morch, A. Opdahl (Eds.), *Proceedings of the 24th IRIS Conference,* 1-14, Ulvik, Department of Information Science, Bergen, Norway.

Guilford, J. P. (1950). Creativity. *American Psychologist*, 5, 444–454. http://dx.doi.org/10.1037/h0063487

Guilford, J. P. (1967). Creativity: Yesterday, today, and tomorrow. *The Journal of Creative Behavior*, 1(1), 3–14. https://doi.org/10.1002/j.2162-6057.1967.tb00002.x

Greer, B., Verschaffel, L. & De Corte, E. (2002). "The answer is really 4.5": Beliefs about word problems. *In Beliefs: A hidden variable in mathematics education?* (pp. 271-292). Springer, Dordrecht. https://doi.org/10.1007/0-306-47958-3_16

Hany, M. & El-Demerdash, M. (2015). "The effectiveness of a proposed unit in bio-mathematics for developing secondary stage students' deep understanding skills - in Arabic," *Journal of Egyptian Association for Science Education, 18*(5). https://doi.org/10.21608/mktm.2015.113172

Haylock, D. (1997). Recognising mathematical creativity in schoolchildren. *ZDM*, 29(3), 68-74. https://doi.org/10.1007/s11858-997-0002-y

Hoyles, C. & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop et al. (Eds.), *Second International Handbook of Mathematics Education*. Dordrecht: Kluwer Academic Publishers. https://doi.org/10.1007/978-94-010-0273-8_11

Hennessey B. A. & Amabile T. M. (1999). Consensual assessment. In M. A. Runco, & S. R. Pritzker (Eds.), *Encyclopedia of Creativity*, 34–36. New York: Academic Press.

Laurillard, D., Stratfold, M., Luckin, R., Plowman, L. & Taylor, J. (2000). Affordances for learning in a non-linear narrative medium. *Journal of Interactive Media in Education,* (2). https://doi.org/10.5334/2000-2

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted, 30*(2), 236-260. https://doi.org/10.4219/jeg-2006-264

Mercat, C. (2014). *D6.1: Report on the creative design process of communauté d'intérêt - Cycle 1. Public report, Mathematical Creativity Squared, FP7 – ICT, Grant Agreement no: 610467.* Accessed 7 April 2025 from: http://mc2-project.eu/index.php/dissemination

Mercat, C. (2015). *D6.2: Report on the creative design process of CoI - Cycle 2*. Public report, Mathematical Creativity Squared, FP7 – ICT, Grant Agreement no: 610467. Accessed 7 April 2025 from: http://mc2-project.eu/index.php/dissemination

Partnership for 21st Century Skills. (2011). P21 common core toolkit: *A guide to aligning the common core state standards with the framework for 21st century skills,* ERIC Clearinghouse. https://files.eric.ed.gov/fulltext/ED543030.pdf

Star, S. L., & Griesemer, J. R. (1989). Institutional ecology, translations' and boundary objects: amateurs and professionals in Berkeley's Museum of Vertebrate Zoology, 1907-1939. *Social Studies of Science*, *19*, 387-420. http://www.jstor.org/stable/285080?origin=JSTOR-pdf

Sternberg, R. J. (2003). The development of creativity as a decision-making process. In R. K. Sawyer, V. John-Steiner, S. Moran, R. J. Sternberg, D. H. Feldman, J. Nakamura, & M. Csikszentmihalyi (Eds.), *Creativity and development*, 91–138. New York: Oxford University Press. https://doi.org/10.1093/acprof:oso/9780195149005.003.0004

Toma, R., Yánez-Pérez, I. & Meneses-Villagrá, J. (2024) Towards a socio-constructivist didactic model for integrated STEM education. *Interchange,* 55, 75–91. https://doi.org/10.1007/s10780-024-09513-2

Torrance, E. P. (1974). *The Torrance tests of creative thinking: Norms-technical manual*. Princeton, NJ: Personal Press. https://psycnet.apa.org/doiLanding?doi=10.1037%2Ft05532-000

Trgalová, J. (2016). *D6.3: Report on the creative design process of CoI - Cycle 3. Deliverable of the MC2* project, FP7 - Information and Communication Technologies, Grant Agreement no: 610467. Accessed 7 April 2025 from: http://mc2-project.eu/index.php/dissemination

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge university press. https://doi.org/10.1017/CBO9780511803932

Abbreviations

CMT: Creative Mathematical Thinking
SC: Social Creativity
MC2: Mathematical Creativity Squared
c-book: Creative Book
CoI: Community of Interest
CoP: Community of Practice.