The Effectiveness of Van Hiele-Based Instruction Supported by Geometer's Sketchpad in Enhancing Student Teachers' Geometry Content Knowledge and Attitudes Towards Technology Integration

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Abstract:

Regarding the role of teacher education in preparing student teachers (STs) to teach geometry to future pupils, this study investigated the effectiveness of Van Hiele-based instruction supported by Geometer’s Sketchpad (GSP) to enhance STs’ Geometry Content Knowledge (GCK) and attitudes towards technology integration through the one-group pretest-posttest design. Accordingly, a sample of sixty STs enrolled in the second year of the undergraduate mathematics education program at the Faculty of Education, Tanta University in Egypt, was selected. Then, the course content was arranged and taught to them through an instruction model designed based on Van Hiele’s theory and supported by the GSP over eight weeks during the academic year 2021-2022, while the GCK test adapted from the TEDS-M and the scale of attitudes were applied before and after the intervention. Overall, results of paired-sample t-tests on the differences between STs’ pre- and post-assessment scores indicated a significant enhancement in STs’ GCK, particularly knowledge of 2-D shapes and related concepts of perimeter and area, in addition to a positive change in their attitudes towards technology integration, especially in the affective component; both with large effect size. As a result, implications for mathematics teacher educators to better prepare STs to teach geometry effectively were discussed.

Keywords: Van Hiele’s theory, Geometer’s Sketchpad (GSP), Geometry Content Knowledge (GCK), Mathematics Teacher Education, Technology Integration

مستخلص البحث باللغة العربية:

ينطلق هذا البحث من ركيزة أساسية وهي الدور الذي تلعبه برامج إعداد المعلم في تأهيل الطلاب المعلمين لتدريس الهندسة لتلاميذهم المستقبليين بشكل فعّال. وبناءً على ذلك، فالبحث الحالي يتقصى فعّالية التدريس القائم على نظرية ڤان هيل والمُدعم ببرنامج Geometer's Sketchpad في تنمية معرفة الطلاب المعلمين للمحتوى الهندسي ورفع اتجاهاتهم نحو دمج التكنولوجيا في التدريس من خلال التصميم البحثي (قبلي- بعدي) مجموعة واحدة. ولتحقيق ذلك، أُختيرت عينة مكونة من عدد (٦٠) طالب معلم من الطلاب المسجلين بالفرقة الثانية ببرنامج إعداد معلمي الرياضيات بكلية التربية، جامعة طنطا والتي تعد واحدة من بين أقدم الجامعات الحكومية المصرية. وتمت إعادة تصميم محتوى مقرر التدريس المصغر ليرتكز على المفاهيم الهندسية الأساسية، ليتم تناول هذا المحتوى من خلال نموذج تعلم مقترح بُنيت مراحله في ضوء مستويات ڤان هيل للتفكير الهندسي ودُعِمت ببرنامج Geometer's Sketchpad خلال (٨) أسابيع متتالية من الفصل الدراسي الثاني لعام ٢٠٢١-٢٠٢٢. وبشكل عام، فقد كشفت نتائج إجراء اختبارt لعينتين مرتبطتين عن وجود فرق ذو دلالة إحصائية بين متوسطي درجات الطلاب المعلمين في التطبيقين القبلي والبعدي لكل من اختبار المعرفة الهندسية ومقياس الاتجاه نحو دمج التكنولوجيا في التدريس لصالح التطبيق البعدي. وبشكل أكثر تحديدا، كان للنموذج التدريسي فعّالية ملحوظة (معدل كسب كبير) في اكتساب الطلاب المعلمين للمعرفة الرياضية المرتبطة بالأشكال الهندسية المستوية ومفهومي المحيط والمساحة، وفي رفع المكون العاطفي المرتبط بالاتجاه نحو دمج التكنولوجيا في التدريس. ونتيجة لذلك، فقد أوصى البحث بضرورة إعادة النظر في محتوى وطرائق تقديم وتقييم المقررات التربوية المُضمنَة ببرامج إعداد معلمي الرياضيات، فالمحتوى يجب أن يرتبط بشكل وثيق بالمفاهيم المطروحة بالمقررات المدرسية بدلا من كونه محتوى عام فارغ من التخصص؛ بمعنى التحول من ثقافة تدريس البيداغوجية pedagogy إلى تدريس معرفة المحتوى البيداغوجية pedagogical content knowledge. أما عن طرائق التدريس فيجب انتقائها بما يتسق مع النظريات التي ثبُتت فعّاليتها تجريبياً في تنمية المعرفة الرياضية للمعلمين، كأن تتم إعادة صياغة وتنظيم الأنشطة التربوية الهندسية في ضوء مستويات التفكير الهندسي لڤان هيل أو أن يتم توظيف البرامج التكنولوجية الهندسية مثل GeoGebra, GSP, Cabri 3D, Euler 3D في التدريس، أو أن يُدمج المدخلين معاً ليحصل الطلاب المعلمين على أقصى استفادة. وبتطوير المحتوى وطرائق تدريسه يجب أن تُرّقى أيضا تكنيكيات التقييم من مجرد كونها اختبارات نمطية قائمة في أغلب الحالات على سرد المفاهيم التربوية، إلى استحداث نماذج وظيفية يمكن من خلالها الكشف عن المهارات التدريسية الحقيقية للطلاب المعلمين. واخيراً، يجب يأخذوا الاخذ بعين الاعتبار محدودية السياق المحدد الذي أُجري فيه البحث الحالي والذي قد يحد من قابلية تعميم نتائجه. ومن ثم، فان هناك ضرورة لإجراء دراسات مماثلة في سياقات اخرى (دول عربية غير مصر) لتقصي مدى فعّالية النموذج المُقترح وكيفية تنقيحه والوصول به لصورة أكثر عمومية تتناسب وإعداد معلمي الرياضيات محليا واقليمياً.

1. INTRODUCTION

The preparation of mathematics teachers, particularly aspects of knowledge, remains of growing concern because of its impact on students learning (Fennema & Franke, 1992; Hill et al., 2004; Stigler & Hiebert, 1999). Teachers' knowledge was initially defined by Shulman (1986) through the categories of subject matter content knowledge (SMK), pedagogical content knowledge (PCK), and curriculum knowledge, wherein the SMK stays essential for the development of other types of knowledge (Brown & Borko, 1992; Leinhardt & Smith, 1985).

Among school mathematics content areas, geometry helps develop students' reasoning, conjecturing, and justification skills (Jones, 2002; National Council of Teachers of Mathematics [NCTM], 2000); these skills should be essentially practiced under the guidance of mathematics teachers. Nonetheless, multiple studies addressing student teachers, pre-service teachers, and novice teachers' knowledge revealed that they are not equipped with the content knowledge needed for teaching geometry (e.g., Aslan-Tutak & Adams, 2015; Browning et al., 2014; Mashingaidze, 2012; Telima, 2011). In other words, prospective teachers are required to teach geometry effectively, yet they might have insufficient geometry content knowledge (Jones, 2000; Sunzuma & Maharaj, 2019). Therefore, to improve students' learning of geometry, it is essential to reinforce those teachers' geometry content knowledge (GCK), which could be done through their preparation program.

A leading framework that supports learning and teaching geometry, precisely learners' geometric thinking, is Van Hiele's model (Hassan et al., 2020; Sinclair et al., 2016). Hence, several studies suggested designing geometry instruction based on the progression of Van Hiele's levels of geometric thinking so that better learning outcomes could be attained (e.g., Armah et al., 2018; Haviger & Vojkůvková, 2014).

Moreover, in teacher education, supporting the instruction with technology was highlighted, not just to overcome such difficulties in learning geometry but also as a necessity to meet the 21st century's skills; thus, categories of teachers' knowledge were expanded to the notion of Technological Pedagogical Content Knowledge (TPACK). It describes an amalgam of teachers' knowledge of content, pedagogy, and technology, which is required to efficiently operate such technology to teach this specific content (Mishra & Koehler, 2006). TPACK also reflects a powerful strategy to train student teachers to incorporate technology in their future teaching of mathematics (Agyei & Voogt, 2011; Zambak & Tyminski, 2020), wherein their attitudes towards technology integration would influence how they are going to implement the technological tools (Belbase et al., 2020). Accordingly, teacher education research revealed that student teachers' knowledge of teaching could be enhanced through content courses that provide opportunities to use mathematics-specific technologies (Niess, 2008; Yigit, 2014); and investigations of their attitudes towards the usage of such technologies in the classroom remains of interest (Mangi et al., 2021).

In geometry instruction, technology plays a crucial role in engaging learners in active learning environments (NCTM, 2000); specifically, dynamic geometry software; it maintains a notable impact on improving understanding of geometric concepts (Baki et al., 2011; Chan & Leung, 2014; Zambak & Tyminski, 2017). Among this software, Geometer's sketchpad (GSP) represents an essential tool for constructing and analyzing mathematical objects wherein learners focus on achieving mathematical objectives instead of how GSP could be operated (Chew & Lim, 2010; Meng & Sam, 2013).

Furthermore, considering the positive effect of GSP on learning and teaching geometry, a trend of research has recommended incorporating it into Van Hiele-based instruction to achieve the expected goals (e.g., Abdullah & Zakaria, 2013; Chew & Lim, 2010; Tieng & Eu, 2018). Still, only a limited number of studies have investigated the GSP effectiveness “through a comprehensive approach and from the perspectives of mathematics teachers’ pedagogical and growth needs in teaching and learning” (Huang et al., p. 100). This is the perspective adopted in the current study, wherein an instructional model designed in light of Van Hiele's theory and supported by GSP has been implemented to enhance student teachers’ GCK and attitudes towards technology integration.

2. PROBLEM STATEMENT AND RESEARCH QUESTIONS

Locally, in several faculties of education, student teachers' lack of mathematical knowledge for teaching was recently highlighted, and recommendations on designing tutorial programs to enhance this knowledge were declared (Abd-Allal, 2017). Yet, there is a scarcity of research that responds to these recommendations (Abd-Alfatah, 2021; Hassan & Al-Raees, 2018), particularly regarding geometry education.

This mirrors the claim that student teachers, during their preparation in the Egyptian context, study general pedagogical courses with no emphasis on specific content (Elbehary, 2020). It is also consistent with Li and Kulm (2008), who expressed the term -double discontinuity-. While one discontinuity describes the gap between secondary school mathematics and university mathematics, the other happens when the student teachers finish college and see the gap between mathematics learned at college and what they are required to teach at the school level.

Accordingly, the necessity to design specific didactic courses at the university level to teach school mathematics content was highlighted (Niyukuri et al., 2020).

Additionally, previous research on technology integration in learning geometry, specifically GSP, has mainly concentrated on school pupils. For instance, at the national level, Al-Meqdadi (2004) used GSP to enhance 9th-grade Jordanian students' understanding of some geometric concepts. Also, Marei (2014) conducted a parallel study wherein teaching geometry through GSP positively affected 7th-grade Jordanian students' learning of geometric transformations. Khaswana and Abu Eraq (2009) reported similar results with 9th-grade Emirati students. More recently, Al-Saedey's (2016) study revealed the significance of utilizing GSP in teaching analytical geometry to improve 9th-grade Saudi students' achievement.

Correspondingly, in the Egyptian context, two studies were found. Mohammad (2020) used GSP to enhance 6th-grade students' geometric sense and visual thinking skills, while Al-Ashrey (2020) employed GSP to overcome visual perception disorders and reduce the mathematical anxiety of 9th-grade students with mathematics disabilities. As a result, these studies recommended training the student teachers on using GSP during geometry instruction because of its positive impact on pupils' learning.

Such a limited number of local investigations that operated GSP, particularly with student teachers, is compatible with recent reviews of (a) Cevikbas and Kaiser's (2021) on dynamic and interactive mathematics learning environments and (b) Ondeş's (2021) on trends in dynamic geometry software. As expressed, GeoGebra was the most preferred software used throughout many studies, and there is a need to consider students' and teachers' lack of experience with other types of software (e.g., Cabri 3D, GSP).

Considering what is raised above, besides the role of teacher education in providing student teachers with effective interventions to promote their understanding of mathematics (López-Martín et al., 2022; Peace et al., 2018; Sánchez, 2011), this study aimed to

design an instructional model based on Van Hiele's theory and supported by GSP software so that student teachers' geometry content knowledge and attitudes towards technology integration could be enhanced.

More specifically, the study addresses the following research questions:

[1] What is the effectiveness of the instructional model based on Van Hiele's theory and supported by GSP in enhancing student teachers' GCK?

[2] What is the effectiveness of the instructional model based on Van Hiele's theory and supported by GSP in enhancing student teachers' attitudes towards technology integration?

3. THEORETICAL FRAMEWORK AND DESIGN OF THE INSTRUCTIONAL MODEL

3.1. Geometry Content Knowledge (GCK)

According to Shulman (1986), the father of research on teachers' knowledge, three categories of teachers' knowledge could be distinguished: curriculum knowledge; PCK; and SMK, which characterizes the basis for other types of knowledge and is the focus of this study. In particular, student teachers must understand the mathematical content to accomplish their future jobs as mathematics teachers (Segarra & Julià, 2021).

Based on Shulman's research, teachers' knowledge required for teaching mathematics was further clarified through the prominent framework of Mathematical Knowledge for Teaching (MKT) (Ball et al., 2008) that described six categories: three under SMK (i.e., common content knowledge, specialized content knowledge, and horizon content knowledge) and three under PCK (i.e., knowledge of content and teaching, knowledge of content and students, and knowledge of content and curriculum). Because of the clarity and applicability of the MKT model, it is widely cited in mathematics education research studies; further to this, it inspired the international Teacher Education and Development Study in Mathematics (TEDS-M). The TEDS-M examined future teachers’ knowledge and beliefs required for teaching mathematics (Tatto et al., 2008) and constituted a basis while outlining the study instruments.

Although geometry characterizes a central content area from the primary level until secondary school mathematics nationally and internationally (The Egyptian Ministry of Education and Technical Education, 2018; NCTM, 2000), student teachers, pre-service teachers, and novice teachers' GCK deficiency were declared in multiple studies (e.g., Abd-Allal, 2017; Adolphus, 2011; Aslan-Tutak & Adams 2015; Segarra & Julià, 2021). To overcome so, strengthening prospective teachers’ GCK became a topic of concern in mathematics teacher education, notably because teachers with inadequate content knowledge have a limited impact on students’ performance (Stigler & Hiebert, 1999). This could be achieved through multiple research approaches; incorporating dynamic geometry software in the Van Hiele-based instruction environment expresses one embraced in this study and is described below.

3.2.Van Hiele’s Theory

Van Hiele's theory designates an outstanding instructional model that aims to improve learners’ levels of geometric thinking (Armah & Kissi, 2018; Hassan et al., 2020; Sinclair et al., 2016). According to it, learners’ geometric thinking progresses through these five levels: Recognition, Analysis, Informal Deduction, Deduction, and Rigor (Van Hiele, 1986; Yi et al., 2020).

In detail, in level 1 (Recognition), learners would be able to recognize, name, and sort geometric shapes (e.g., triangles, parallelograms, rectangles) based on their physical appearance. In level 2 (Analysis), learners would be capable of analyzing and differentiating these shapes based on their properties; and they could use the appropriate terminology to describe them. Further, level 3 (Informal Deduction) could be accomplished when learners perceive relationships among geometric shapes; continuously when they comprehend the role of deduction in formulating theorems and proofs, they move to level 4 (Formal Deduction). Finally, in level 5 (Rigor), learners can work in an axiomatic abstract system (Abdullah & Zakaria, 2013; Alex & Mammen, 2018; Tieng & Eu, 2018; Van Hiele, 1986).

Along with these levels, Van Hiele-based instruction was proposed wherein the cognitive progress in geometry could be accelerated (Van Hiele, 1986). It describes an effective practice to overcome the challenges of teaching and learning geometry (Armah et al., 2018; Mostafa et al., 2017; Ramlan, 2016). Van Hiele-based instruction is determined by five sequential phases of learning: Information, Guided Orientation, Explicitation, Free Orientation, and Integration (Crowley, 1987; Naufal et al., 2020; Vojkuvkova, 2012). These phases constituted a foundation while designing the instructional model employed in this study (see Figure 1).

3.3.Geometer’s Sketchpad and Attitudes Towards Technology Integration

Technology plays an essential role in geometry teaching and learning wherein “tools such as dynamic geometry software enable students to model, and have an interactive experience with, a large variety of two-dimensional shapes” (NCTM, 2000,

p. 41). Moreover, technology integration during teacher preparation symbolizes a challenge for developing countries, wherein most research on dynamic geometry environments, specifically, was conducted in developed country contexts (Ndlovu et al., 2013).

In this regard, teacher education programs should inspire student teachers to incorporate the appropriate technological tools into their future teaching; alternatively stated, change their mindsets towards utilizing technology in mathematics instruction (Belbase et al., 2020; Chai et al., 2011).

Yet, the effectiveness of integrating technology in the classroom does not merely rely on teachers' knowledge of operating the technological tools, but also on their attitudes. As articulated by Teo, the “success of any initiatives to implement technology in an educational program depends strongly upon the support and attitudes of teachers involved” (2008, p. 414). Thus, exploring the student teachers’ attitudes towards technology integration was underlined in this study, wherein it is crucial to predict a productive implementation of such technology in future classrooms (Huang & Liaw, 2005; Mangi et al., 2021). Also, it is measured upon Selwyn's (1997) model that determines the attitude through four components: affective, perceived usefulness, perceived control, and behavioral intention (Meng, 2012; Teo, 2008).

Through dynamic geometry software, learners could create figures by dragging them; accordingly, they could make estimations, examine possible motions, discover patterns, formulate mathematical phrases, justify ideas, and write geometric proofs (Christou et al., 2004). More precisely, operating GSP was emphasized to enhance learning geometry (e.g., Chew & Lim, 2010; Ganesan & Eu, 2020; Hartono, 2020; Kesan & Calsiskan, 2013; Meng & Sam, 2013; Uygun, 2020).

GSP describes one type of dynamic geometry software through which learners can easily construct and manipulate the geometric objects on the screen, wherein these objects stay coherent whenever they are dragged so that further analysis of their properties can be done (Furner & Marinas, 2007; Uygun, 2020). Also, integrating GSP into the proposed model admits Ruthven’s (2008) argument regarding the importance of dynamic geometry, wherein “the development of ideas was organized around task-focused use of the software by students, structured and shaped by the teacher in ways considered beneficial for building mathematical knowledge” (p. 382).

Furthermore, and more relevant to the context of this study, multiple researchers conveyed the significance of integrating GSP into Van Hiele-based instruction compared to conventional approaches. For example, Abdullah and Zakaria (2013) and Chew and Lim (2010) indicated how this integration affects students’ levels of geometric thinking; while the former developed activities on the topic of transformations, the latter focused on concepts of the equilateral triangle, square, regular pentagon, and regular hexagon. Similarly, Tieng and Eu (2018) declared the impact of phase-based instruction utilizing GSP on developing primary school pupils’ Van Hiele levels of geometric thinking regarding the concept of angle. This corresponds to the conclusion of Hassan et al.’s (2020) review on students’ geometric thinking since they reported that studies on technology-based intervention, specifically of GSP combined with the van Hiele phases, had a very large effect size.

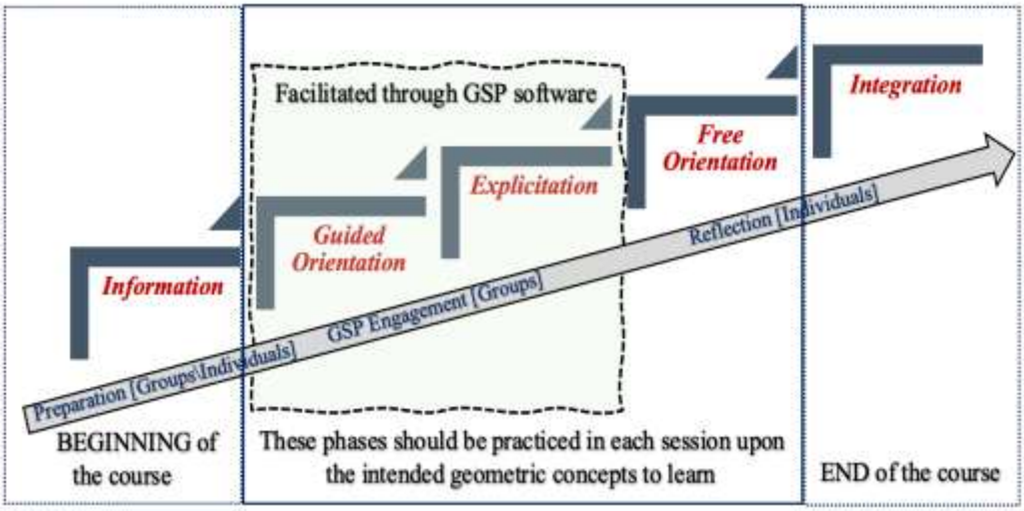
As noticed, regardless of research evidence on the Van Hiele-based instruction supported by GSP, most of this research was primarily intended to promote students’ geometric thinking, not teachers' knowledge. This could be complemented by what was uncovered while reviewing recent local studies on mathematics teacher education, wherein few investigations have concentrated on enhancing student teachers'[I]knowledge (see the Problem Statement section).

3.4. The Instructional Model Design (the treatment)

Considering the theoretical grounds discussed above, the instructional model acknowledged in this study is exhibited in Figure 1, followed by the practices of the instructor upon stages of this model (Crowley, 1987; Drijvers et al., 2010; Haviger & Vojkůvková, 2014; Tieng & Eu, 2018; Van Hiele, 1986; Vojkuvkova, 2012).

Figure 1

An instructional model based on Van Hiele's theory and supported by GSP



Phase 1 [Information]. It specifies the preparation stage wherein oral discussions on geometric concepts are conducted. During this phase, the instructor should: (1) clarify STs' prior knowledge of the intended geometric concepts through warm-up questions, for instance: When do pupils first learn this concept

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[I] **Henceforth, the term "student teachers" will be replaced by the abbreviation STs** by the school curriculum? Why do you think this concept is valuable to acquire? How could you define this concept to your future pupils? What are geometric shapes associated with this concept (draw some examples)? (2) Allow STs to form their groupwork comprising three participants at least so that each group uses one shared computer. (3) Invite STs to respond to pre-assessments of the GCK test and scale of attitudes towards technology integration.

Phases 2 and 3 [Guided Orientation and Explicitation]. It determines the GSP engagement stage (i.e., engagement in interactive geometric activities on GSP). During this phase, the instructor should: (1) Guide STs to draw several initial models of the geometric concept through GSP in light of their prior knowledge, accordingly, discussing the common characteristics throughout these models so that STs can infer the concept definition and describe it in their own words. (2) Emphasize the related technical terms to the concept (e.g., diagonal, center, height, parallel). (3) Specify the necessary and sufficient conditions of the concept. (4) Underline the difference between the definition of the concept from its properties. (5) Draw a standard set of examples and non-examples through GSP.

Phases 4 and 5 [Free Orientation and Integration]. It defines the reflection stage wherein the STs are engaged in reflective discussions and individual geometric proof tasks. During this stage, the instructor should: (1) Engage STs in working on more complex tasks that involve geometric proof, for example, employ the deductive proof to verify the properties of a geometric concept through its definition; or judge whether a given description could be a definition for a geometric concept or not? (2) Provide STs with the required feedback on their group and individual works by sharing and discussing their common misconceptions. (3) Help STs to synthesize what they understood about the geometric concepts through a clear overview. (4) invite STs to respond to the post-assessments.

4. METHOD

4.1.Research design

The current study was framed upon the pre-experimental design in which “the researcher studies a single group and provides an intervention during the experiment. This design does not have a control group to compare with the experimental group” (Creswell, 2009, p. 158). Specifically, the *One-Group Pretest-Posttest Design,* which includes a pretest measure followed by a treatment and a posttest for a single group, was employed to examine whether there was a significant difference in STs’ GCK and attitudes towards technology integration before and after implementing the proposed instructional model.

The one-group design used in this research has received criticism due to factors like maturation and test effects (Marsden & Torgerson, 2012). Yet, it was selected in order to avoid any potential harm that could arise from assigning participants randomly (ethical concern). In other words, all STs enrolled in the second year of the mathematics teacher education program at the faculty of education during 2021-2022 were treated equally in terms of the course content proposed in this study.

4.2.Participants and Context

The study sample consisted of sixty STs who were selected through the *Judgmental Sampling Strategy* (Taherdoost, 2016); that is, among the sixty-eight STs enrolled in the second year of the undergraduate mathematics education program at the Faculty of Education, Tanta University, Egypt, during the second semester of the academic year 2021-2022, sixty were specified. Those participants were orderly attending lectures and completed all the instructional activities of this study; in other words, they represented the sample warranted inclusion (Taherdoost, 2016).

According to the faculty curriculum, during the second year of the preparation program, STs must join two compulsory courses micro-teaching and educational technology. While the former aims to enhance their knowledge of school mathematics and train them to encounter the teaching practicum, the latter seeks to prepare them to employ technology in their future profession. Considering this, the present study was conducted during the practical sessions of the information technology course, given the overarching common goal of developing STs' TPACK.

4.3.Instruments

Two instruments were employed in this study (see Appendix I): (A) the GCK test, which was adapted from the TEDS-M (Brese & Tatto, 2012) that stands as the most influential international investigation on pre-service teacher education (Segarra & Julià, 2021). (B) the scale of attitudes towards technology integration, which was constructed upon Selwyn's (1997) model, and its statements were paraphrased based on multiple relevant studies that concentrated on technology integration for samples of pre-service teachers (e.g., Mangi et al., 2021; Meng, 2012; Teo, 2008).

The GCK test consisted of nine multiple-choice items with different scores depending on (a) the number of minor questions included in each item and (b) whether this item requires progressing into steps to get the answer. As detailed in Table 1, items 2, 3, and 5 are scored 0 or 1; item 6 (two minor questions) is scored 0, 1, or 2; item 9 (three minor questions) is scored 0, 1, 2, or 3; and items 1, 4, 7, and 8 are scored considering the steps executed by the student-teacher (see the Results section).

Accordingly, the whole GCK test was scored on a scale from 0 to 19.

Table 1

Characteristics of GCK test (Adapted from TEDS-M)

Items	*Points*	*Cognitive domain*	*Concepts embedded*	*The answer key (correct answer)*
1.	**3**	**Applying**	**Area of rectangles**	**No. 3 (Scale)**
2.	**1**	**Knowing**	**Special cases of parallelograms**	**No. 3**
3.	**1**	**Knowing**	**Volume of cubes**	**No. 1**
4.	**3**	**Applying**	**Area of triangles and squares**	**Scale**
5.	**1**	**Knowing**	**3-d folded figures**	**No. 4**
6.	**2**	**Knowing**	**Solution of an equation in a plane and in space**	**No. 2 (plane) and No. 3 (space)**
7.	**3**	**Applying**	**Perimeter, parallelogram, triangle**	**Scale**
8.	**2**	**Reasoning**	**Perimeter, cylinder, cube**	**Scale**
9.	**3**	**Applying**	**Lines of symmetry of regular hexagon, pentagon, and rhombus**	**No. 1 (hexagon), No. 1 (pentagon), and No. 2 (rhombus)**
Total	**The test of 9 items and its highest score equals 19 points**			

On the other side, the scale of attitudes towards technology integration included twenty-two statements, which are categorized into four components: affective (from 1 to 4), perceived usefulness (from 5 to 10), perceived control (from 11 to 16), and behavioral intention (from 17 to 22). As shown in Table 2, all positive items were measured upon a five-point Likert scale of strongly agree (5), agree (4), neutral (3), disagree (2), and strongly disagree (1), while negative items were measured reversely. Finally, the scores of each component were aggregated to calculate each student teacher's score per component.

Table 2

Characteristics of the scale of attitudes towards technology integration

Items	*Positive items*	*Negative items*	*Subcomponent*
1-4	**1, 2**	**3, 4**	**Affective**
5-10	**5, 6, 7, 8, 9, 10**		**Perceived usefulness**
11-16	**12,**	**11, 13, 14, 15, 16**	**Perceived control**
17-22	**17, 18, 21**	**19, 20, 22**	**Behavioral intention**
Total	***12***	***10***	***4 components***
	The scale of 22 items and its highest score equals 110 points		

Moreover, to determine the validity of these instruments (i.e., content validity and translation validity), they were evaluated in terms of items purposes, clarity, and language by three experienced local educators in mathematics education (Taherdoost, 2016); accordingly, the instruments were revised and modified considering their comments and recommendations. Besides, Pearson’s correlation (test r-test stability technique) and Cronbach’s alpha (internal consistency) coefficients were calculated to confirm the reliability of the GCK test and the scale of attitudes, respectively. This was executed during a preliminary investigation involving fifteen STs and aimed at experimenting with the validity and reliability of the instruments. As a result, values 0.70 and 0.73 were obtained for Pearson's correlation and Cronbach’s alpha coefficients, respectively; each indicated an acceptable level of reliability (Koo & Li, 2016; Taber, 2018).

4.4. Processes of the implementation

The course of study was taught over eight weeks (eight sessions), including pre-and post-assessments, wherein each session lasted two and half hours. The content of these sessions was systematized considering the instructional model displayed in Figure 1. This is detailed as follows:

Phase 1 [Information]

- *Duration:* This phase was practiced in the first two sessions of the course [5 hours].

- *Phase theme*: teacher-centered; traditional without GSP; oral group discussions and individual pre-assessments.

During Session 1 (the 1st week of the course), (a) The course outline and objectives were introduced to STs by the instructor (the researcher), who left them on their own to explore the GSP software during the first 30 minutes of the session. (b) STs were invited to organize themselves into tutorial groups, wherein each group included at least three participants so that one computer could be shared. (c) The mathematics-intended curriculum at the elmentary level was reviewed by these groups in which each group focused on a specific grade (4-7) to determine the basic embedded geometric concepts. (d) The deduced concepts were shared among all groups; hence, commonalities were observed. (e) These concepts were systematized under four sessions characterizing the course content. *In Session 2, (a)* STs were asked to answer the GCK paper test and complete the online form of the scale of attitudes towards technology integration. (b) A brief description of GSP functions of the title bar, menu bar, sketch plane, and toolbox; also, its essential tools (selection arrow, point, compass, straightedge, text, and custom tools) were provided.

Phase 2, 3, and 4 [Guided Orientation, Explicitation, and Free Orientation]

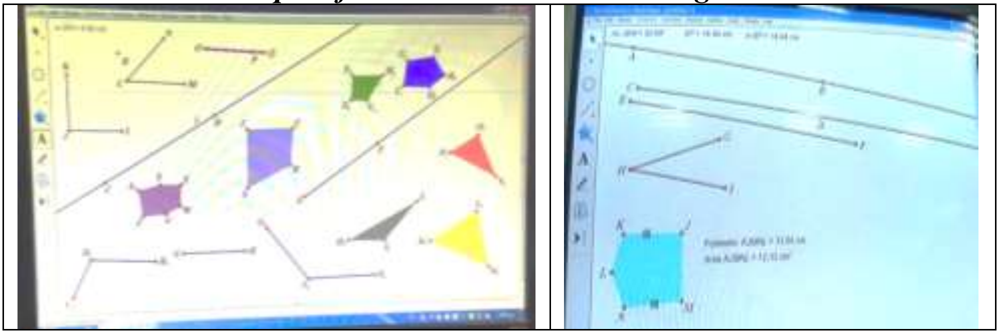
- *Duration*: These phases were practiced altogether for four consecutive sessions [4 weeks].
- *Phases theme*: Phase 2: teacher-centered, facilitated by GSP, oral-group discussions. Phase 3: student-centered, facilitated by GSP, oral-group discussions. Phase 4: student-centered, traditional without GSP, individual written tasks.

In Session 3 (3rd week), the 1st lesson [***Essentials***] was practiced. It emphasized the geometric concepts of the line segment, ray, straight line, and angle (acute, right, obtuse, and straight angle) through these objectives: (a) Draw several line segments, rays, and straight lines through GSP (see Figure 2). (b) Explain relationships among the line segment, ray, and straight line. (c) Define the line segment, ray, and straight line, rigorously. (d) Draw multiple angles with different measures through GSP. (e) Define the concept of angle precisely.

In Session 4 (4th week), Lesson 2 [***Curves and Polygons***] was conducted wherein concepts of the curve (open, closed, simple), circle, polygons, triangle (isosceles, equilateral, and scalene triangle), quadrilaterals, perimeter, and area were discussed. It aimed at achieving these objectives: (a) Draw multiple curves and polygons through GSP (see Figure 3). (b) Determine the vertices, sides, and diagonals of any polygon. (c) Define concepts of curves and polygons, including their different types, precisely. (d) Induce the formula of the sum of interior angles of a polygon. (e) Recognize that the perimeter is measured in linear units while the area is measured in square units. (f) Describe the procedure for finding the perimeter and area of a polygon through GSP.

Figure 2

Sample of STs' works on GSP during session 3



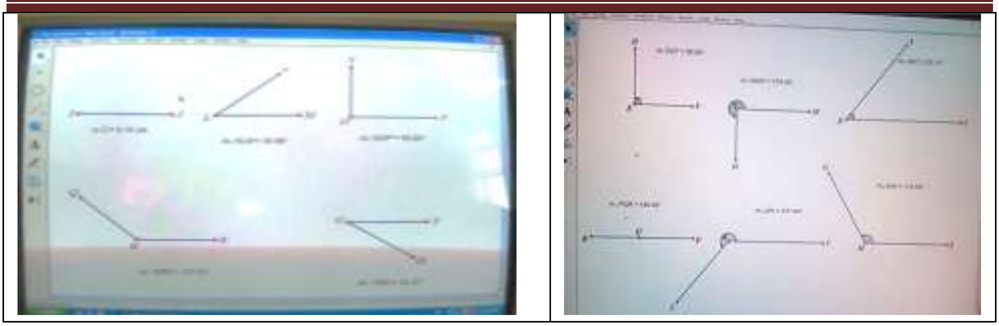
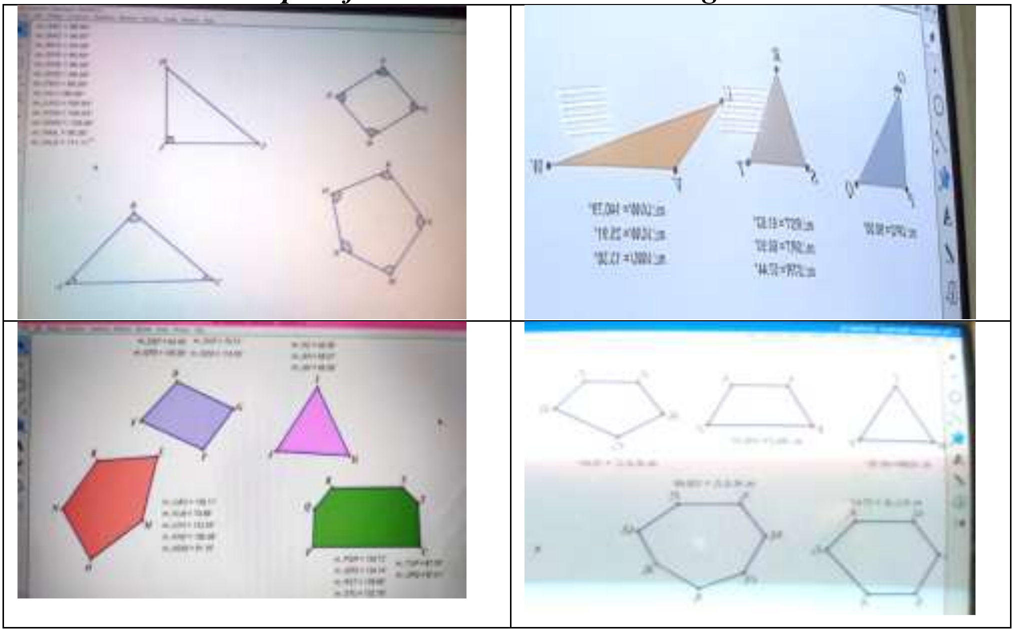


Figure 3

Sample of STs' works on GSP during session 4

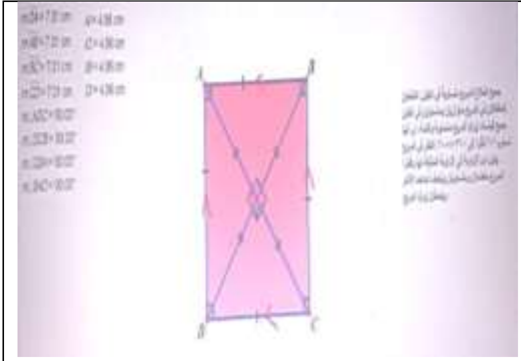
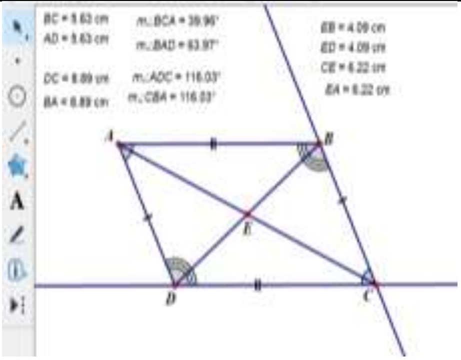
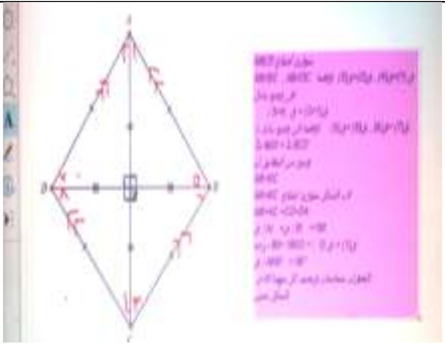
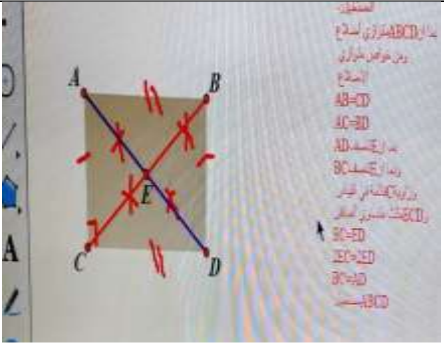


In Session 5 (5th week), Lesson 3 [***Special Parallelograms***] was performed wherein concepts of perpendicular, parallel, trapezium, parallelogram, rectangle, rhombus, square, and line of symmetry were spotlighted through these objectives: (a) Draw several models of quadrilaterals through GSP. (b) Define the concept of the parallelogram as a distinctive quadrilateral. (c) Verify the properties of the parallelogram (relation between its opposite sides, angles, and diagonals) considering its definition.

(d) Illustrate the relationship between the parallelogram and its special cases. (e) Draw examples of trapezium, rectangle, rhombus, and square through GSP (see Figure 4). (f) Affirm properties of trapezium, rectangle, rhombus, and square by manipulating them on GSP, then verifying them deductively. (g) Determine the number of lines of symmetry of special parallelograms.

Figure 4

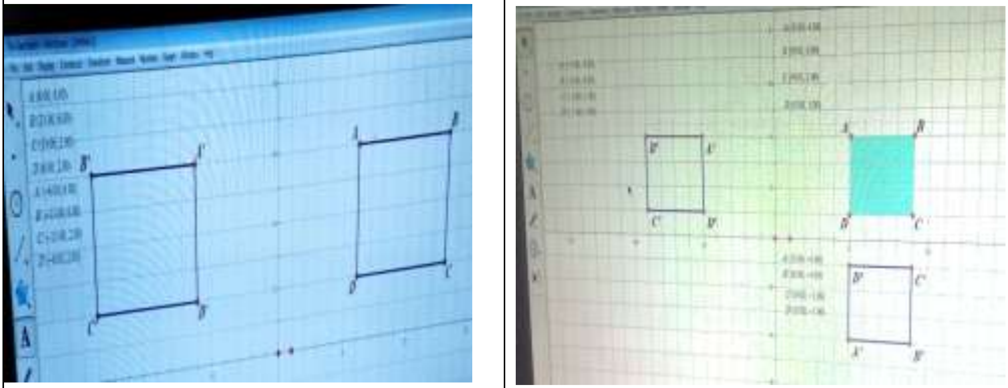
Sample of STs' works on GSP during session 5

 The case of square	 The case of parallelogram
 The case of rhombus	 The case of rectangle

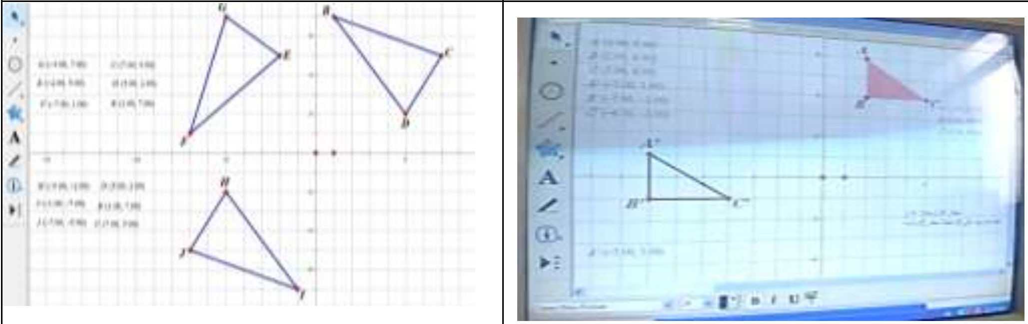
In Session 6 (6th week), Lesson 4 [***Transformations***] was practiced. It strengthened concepts of reflection, translation, and rotation through these objectives: (a) Describe the different types of transformations, including reflections, translations, and rotations. (b) Represent models of transformations, translations, and rotations in the coordinate plane on GSP (see Figure 5). (c) Infer the properties of reflections, translations, and rotations by manipulating several models on GSP. (d) Determine the line of reflection of a segment as the perpendicular bisector (i.e., line of symmetry). (e) Specify the sequence of multiple transformations required to transform a given figure into the other.

Figure 5

Sample of STs' works on GSP during session 6



The reflection across the X-axis | The reflection across X- and Y-axes



The case of rotation | The case of transition

Phase 5 [Integration]

- *Duration:* This phase was practiced during the last two sessions of the course [5 hours].
- *Phase theme*: student-centered; traditional without GSP; individual written tasks and post-assessments.

During Session 7 (7th week), (a) A summary of the course content was introduced. (b) STs were invited to share the difficulties they encountered regarding the geometric concepts discussed during the course and the application of these on GSP; consequently, feedback was given. (c) STs were cordially asked to document their thoughts on the strengths and challenges of obstacles GSP to teaching geometric concepts to their future pupils (see the Results section).

In Session 8 (8th week), STs were asked to answer the GCK paper test and fulfill the technology integration scale, remarking that the sequence of post-assessment items was randomized to reduce the influence of the practice effect.

5. RESULTS AND DISCUSSION

To answer the first research question (*What is the effectiveness of the instructional model based on Van Hiele's theory and supported by GSP in enhancing STs' GCK*?), the paired samples t-test was performed. As a result, the difference between the mean scores was statistically significant at $p < .05$, indicating that STs' GCK was enhanced significantly after teaching the geometry course through the proposed model (see Table 3). Also, to determine the effect size of the intervention on STs' GCK, the point biserial correlation coefficient (r_{pb}) was calculated (Fritz et al., 2012); thus, its value was 0.79, which implies that the employed model has largely influenced STs' GCK (Cohen, 1988, as cited in LeBlanc & Cox, 2017).

Table 3

Paired-samples t-test results of STs' GCK before and after the intervention

	M	*SD*	*df*	*t-value*	*Sig.*	*Effect size (r_{pb})*
Post (N=60)	**14.3083**	**2.21147**	**59**	**9.959**	**.000**	**0.79**
Pre (N=60)	**10.583**	**2.9114**				

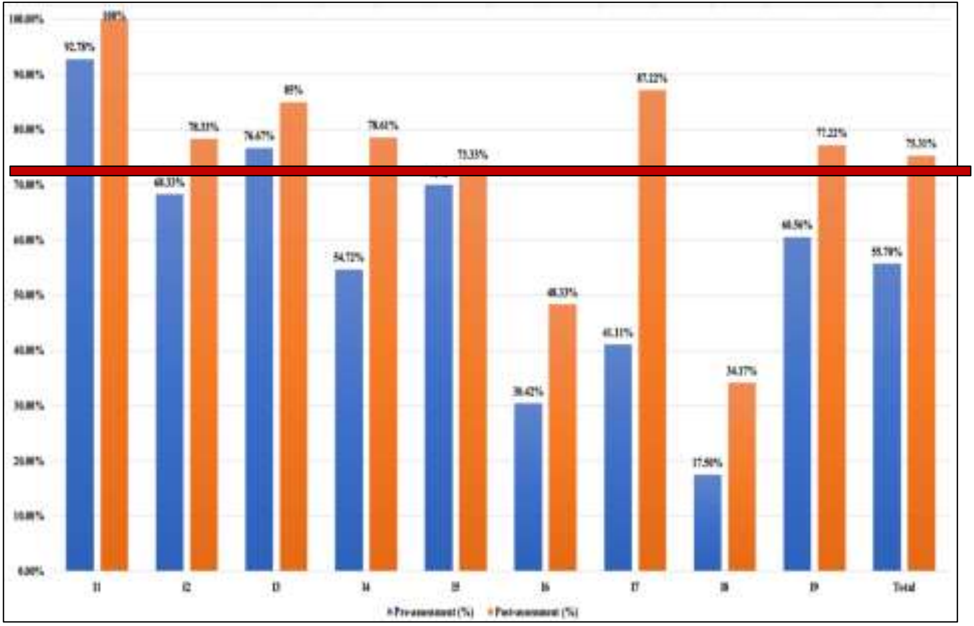
In addition to performing the paired samples t-test and for more clarification concerning the changes of STs' GCK, Figure 6 was represented. It displays changes in mean percentages of STs' attainment of the GCK embedded in items of pre-and post-test.

Considering that 70% characterizes the pre-determined acceptable level of attainment decided in terms of the faculty grading system as equivalents to the grade good (70%-79%), STs' knowledge of I1 (92.78%), I3 (76.67%), and I5 (70%) was adequate before the intervention. On the contrary, STs successfully answered all the post-test items with this acceptable level, excluding I6 (48.33%) and I8 (34.17%). This reflects that even after the intervention, STs’ knowledge of (a) the solution of an equation in a plane and space (I6) and (b) the application of the perimeter concept (I8) remained low.

In detail, regarding I6, some STs correctly selected one line as the solution of a linear equation in the plane; nonetheless, when it came to the space, they could not decide which was the correct answer. Perhaps this happened because neither the proposed course content in this study nor the STs’ prior experience with the school curriculum refers to such an idea. Although pupils start to learn how linear equations can be solved from grade 5, the representation of its solution in a plane is explained in the lower-secondary curriculum (grade 9). Also, the school curriculum is no discussion on what such a solution looks like in space.

Figure 6

Comparison of mean percentage of correct answers of GCK pre-and post-test items

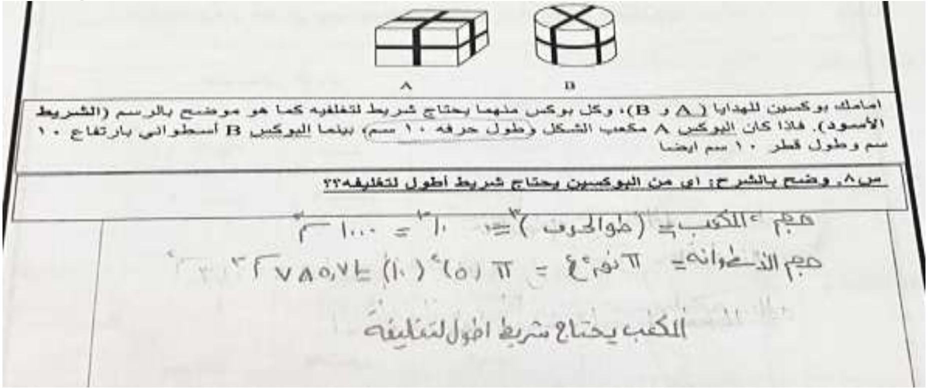


In addition, STs’ answers to I8 were also disappointing since they could not specify what concept (i.e., perimeter, area, volume) is applicable to determine the box with the longer ribbon (see Appendix I). As revealed, most of the STs' wrong answers (score 0) involved comparing the cube volume (1000cm^3) with the cylinder volume (785,71 cm^3) to determine which ribbon is longer; accordingly, they decided that the cube would need a longer ribbon. On the other side, among the few cases of STs who correctly specified utilizing the perimeter as a solution (see Figure 7), some used the cylinder diameter (10cm) instead of its radius to calculate the ribbon length; those were given a score of 1.5 (instead of 2) as a partially correct answer.

Figure 7

Sample of STs' answers to I8

wrong answer (score 0)



أمامك بوكسين للهدايا (A و B)، وكل بوكس منهما يحتاج شريط لتغليفه كما هو موضح بالرسم (الشريط الأسود). فإذا كان البوكس A مكعب الشكل وطول حرفه ١٠ سم) بينما البوكس B أسطواني بارتفاع ١٠ سم وطول قطر ١٠ سم ايضا

س٨. وضح بالشرح: اى من البوكسين يحتاج شريط أطول لتغليفه؟؟

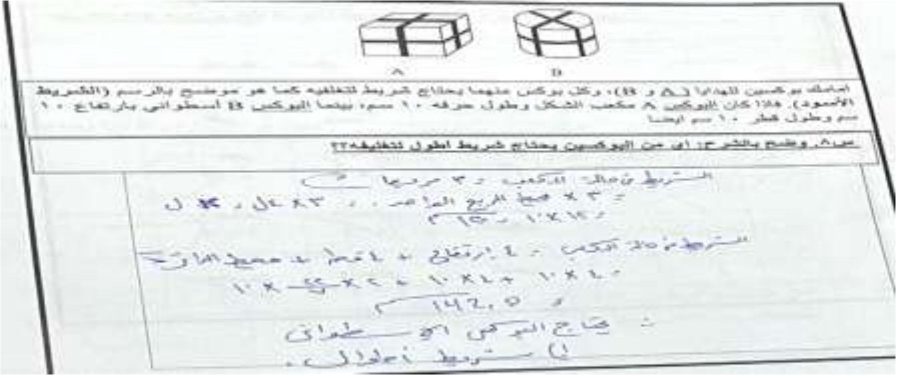
حجم الكعب = (طول الحرف)^٣ = ١٠^٣ = ١٠٠٠ سم^٣

حجم الأسطوانة = ٢٢/٧ نق^٢ ع = ٢٢/٧ (٥)^٢ (١٠) = ٧٨٥,٧١ سم^٣

الكعب يحتاج شريط اطول للتغليفة

Since the cube volume = $L^3 = (10)^3 = 1000cm^3$, and the cylinder volume = $\pi r^2 h$ = 22/7 $(5)^2$ (10) = 785.71 cm^3; then, the cube would require a longer ribbon.

Partially correct answer (score 1.5)



For the cube, each face requires 20 cm, then the whole cube requires 20*6 = 120 cm; while for the cylinder there are (a) 2 circles each of 2*10cm, (b) 4 highs each of 10, and (c) the middle ribbon of 2πr = 4*10 + 4*10 + 2*22/7*10 = 80 +62.9= 142.9 cm. Then, the cylinder would need a longer ribbon.

The previous deficiency could be interpreted in terms of STs' experience to involve the concept of volume when encountering a problem containing 3-D shapes; that is, the term volume mostly comes to their minds. In that sense, arguing concepts of perimeter and area within the context of 2-D shapes and, apart from encountering 3-D shapes (including volume), might not offer STs enough knowledge to discriminate situations wherein such concepts could be applied. In other words, comprehending concepts of perimeter and area would be enhanced through exploring 3-D shapes that are usually addressed in the curriculum by lateral and total surface area and volume.

On the other side, Figure 6 also depicts that STs’ GCK of I4 and I7 was enhanced remarkably after the intervention, wherein the mean percentages of STs’ attainment of the GCK rooted in these items were increased substantially by about 24% and 46%, respectively. Surprisingly, and as opposed to the above analysis of STs' answers to I8, applying the concepts of the perimeter (I7) and area (I4) by STs seemed unmistakable. This could be diagnosed considering two issues. One is its consistency with what was exposed concerning STs’ familiarity with operating these concepts within the context of 2-D shapes (the case of I4 and I7); consequently, the need to incorporate the 3-D shapes to enrich the discussion of the perimeter, area, and volume as interrelated concepts. Second, it might indicate that the intervention enhanced STs’ procedural knowledge of perimeter and area (I4 and I7) compared to conceptual knowledge (I8).

Another possible related aspect is that STs’ knowledge of applying concepts of perimeter and area of 2-D shapes requires an understanding of these shapes themselves (square, triangle, parallelogram), which seemed enhanced in this study through the proposed model. This was clarified by Huang et al. (2020) as the effectiveness of GSP compared to conventional teaching.

While the parallelogram could be created in GSP by constructing the midpoint of each side of a quadrilateral, and then connecting these four midpoints consecutively; this proposition is usually presented in traditional classrooms at first as a fact. Thus, GSP helps learners uncover properties of geometric shapes by themselves rather than merely providing such properties as validities, which matches Armah and Kissi's (2018) argument on the importance of carefully designed activities in guiding learners to test the properties of geometric figures empirically. It also facilitates the process of visualization that reinforces teaching and learning geometry (Duval, 2013), especially if STs are starting from the visualization level of geometrical thinking, wherein they deal with visually presented geometric terminology better compared to verbal ones, as concluded in Alex and Mammen’s (2018) study.

Ultimately, the enhancement of STs' GCK exposed in this study might be generated essentially by the integration process, integrating GSP into the Van-Hiele-based instruction. On the one hand, GSP software worked as a cognitive tool to scaffold geometry learning; then, it enabled STs to attain their zone of proximal development. This reflects Zambak and Tyminski’s (2020) and Huang et al.’s (2020) arguments regarding the significance of dynamic geometry software; besides, it matches Hartono’s (2020) investigation that revealed the value of operating GSP while teaching 2-D geometric shapes.

On the other hand, Salifu et al. (2018) declared that designing the instruction, including activities, in terms of Van Hiele levels helps overcome the conventional ineffective geometry teaching by strengthening the process of concept formation. Also, within the context of teacher education, Yi et al. (2020) reported that the instructional activities developed based on Van Hiele’s theory positively affect pre-service teachers’ GCK.

Similarly, the paired samples t-test was executed to respond to the second research question (*What is the effectiveness of the instructional model based on Van Hiele's theory and supported by GSP in enhancing STs' attitudes towards technology integration?),* and its results are shown in Table 4. These results indicate that the difference between the mean scores of the pre- and post-assessments was statistically significant at p < .05, which reflects that, overall, STs' attitudes towards technology integration have improved significantly after teaching through the proposed model. Moreover, the influence of this model on STs' attitudes was very large, as conveyed by the effect size coefficient value of 0.87.

Table 4

Paired-samples t-test results of STs' attitudes towards technology integration before and after the intervention

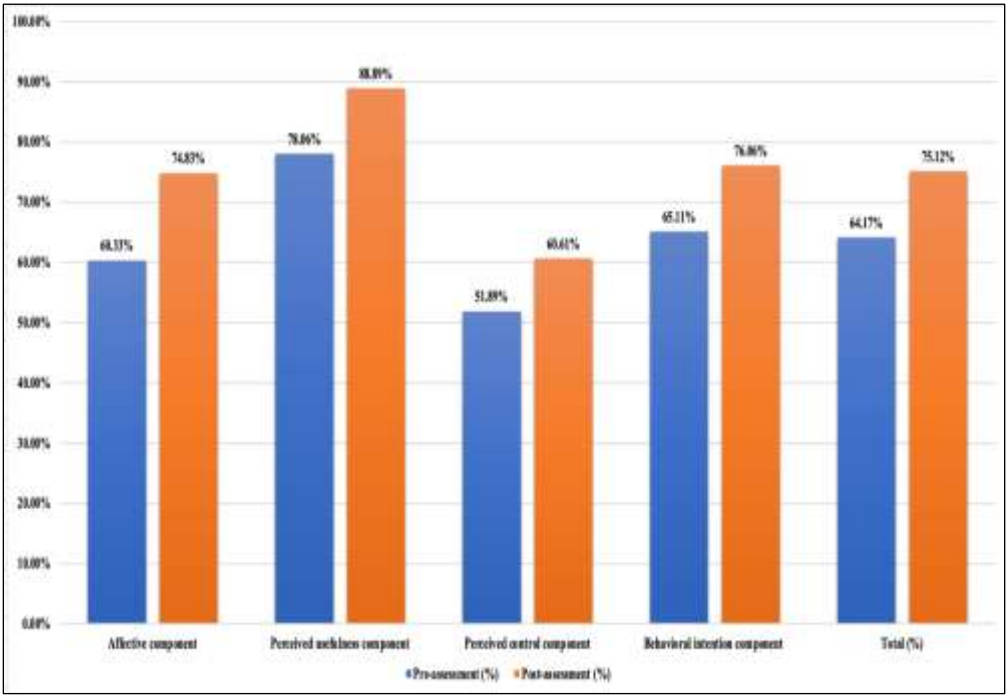
	M	*SD*	*df*	*t-value*	*Sig.*	*Effect size (r_{pb})*
Post (N=60)	**82.6333**	**8.57661**	**59**	**13.447**	**.000**	**0.87**
Pre (N=60)	**70.5833**	**7.14331**				

In detail, considering that STs' attitudes were measured in terms of the affective, perceived usefulness, perceived control, and behavioural intention components (Selwyn, 1997); they persisted in scoring the highest level on the *perceived usefulness* and the lowest on the *perceived control* not only before the intervention (78.06%, 51.89%) but also after it (88.89%, 60.61%) (see Figure 8). This signifies that STs' felt themselves to be in control of technology to a lower degree than they thought the technology is beneficial (perceived usefulness), enjoyable (affective), and should be integrated into future classrooms (behavioural intention).

A comparable result was documented in Teo's (2008) and Mangi et al.'s (2021) studies that examined pre-service teachers' attitudes towards the use of computers in mathematics and revealed that they achieved the lowest on the perceived control component.

Figure 8

Comparison of mean percentages of the pre-and post- scale of attitudes towards technology integration



Moreover, the mean percentage of STs' affective component substantially enhanced after the intervention (from 60.33% to 74.83%) compared to other components that improved by about 10%. This implies that supporting Van Hiele-based instruction by GSP provided STs' confidence and filled them with enthusiasm to integrate technology into their future classrooms since STs were able to realize how geometric concepts could be further discussed and perceived through the assistance of GSP.

Such a conclusion was previously spotlighted by Hartono (2020) and Ganesan and Eu (2020) as they identified technology as a great motivation tool to acquire geometric concepts through which learners feel excited, less stressed, and more enjoyable while the instruction style differs from the traditional manner.

In addition to the previous analysis of STs' responses to the scale of attitudes and to get more insights into how incorporating GSP into the Van Hiele-based instruction positively affected STs' attitudes, they were asked to document their views on the strengths and obstacles of using GSP to teach geometric concepts to their future pupils (see Tables 5 and 5).

As outlined in Tables 5 and 6, STs regarded the strengths and obstacles of using GSP from these three aspects: learning, teaching, and management. When they wrote about the positives, the perceived usefulness was apparent; that is, how teaching geometry through GSP could help learners achieve both cognitive and affective objectives and, at the same time, keep facilitating the teaching process.

On the other side, the management aspect was strengthened in STs' views on the barriers of using GSP, particularly the limited time of its free version and troubles of both hard and software. This coincides with what was revealed before since STs' perceived usefulness of using GSP was the highest while their perceived control of a GSP-based environment was the lowest, which would influence their intended actions of utilizing technological tools in the future teaching of geometry (Belbase et al., 2020).

Table 5

STs' views on the strengths of using GSP

GSP benefits for	*Responses* **Using GSP in geometry education**
Learning (Pupils' side) *37 responses*	• Facilitates learners' access to concepts in an easy, precise, simple, and modern way (13 responses). • Help deduce, accordingly, memorize geometric concepts and maintain this knowledge for a long time (Learning retention) (7). • Attracts learners' attention and concentration by motivating them to learn and making the learning process more enjoyable (7). • Enables learners to understand concepts and geometric shapes

	clearly, deeply, and in a short time compared to traditional methods (5). • Enable learners to compete in the era of technology (4). Help imagine and visualize the drawn geometric figures (1).
Teaching (Teachers' side) *9 responses*	• Keeps teachers' time and effort since it works as a visual tool that facilitates and fosters the instruction process (9 responses).
Management(Environment) *14 responses*	• Precise and unmistakable in drawing and measuring lengths and angles (5 responses). • Cultivate classroom discussions, interaction, and collaboration between teacher and pupils (4). • Easy and simple to use (4). Has a small size compared to other software (1).
Total of 60 responses	

Table 6
STs' views on the obstacles of using GS

GSP obstacles in	***Responses*** **Using GSP in geometry education**
Learning (Pupils' side) ***2 responses***	• **Limits learners' manners of thinking (1 response)** **Limits learners' abilities to draw geometric shapes compared to traditional geometric tools (1).**
Teaching (Teachers' side) ***11 responses***	• **Requires teachers' mastery of the programs to deliver the knowledge to pupils, them more time and training (7 responses).** • **Teachers might make mistakes while using it; or be unable to control either software or hardware troubles (4).**
Management(Environment) ***24 responses***	• **The free version of GSP stays for only 20 minutes (11 responses).** • **Software and hardware problems might be happened suddenly while using (6).** • **Requires more time to be applied and to moderate the classroom discussion (4).** • **Schools are not equipped to teach through similar software (2).** • **The assessment stays traditional and not technology-based (1).**
Total of 37 responses	

6. CONCLUSION AND RECOMMENDATIONS

Admitting the importance of enhancing prospective teachers' GCK and attitudes towards technology integration, besides the lack of research in this area at a local level, the current study employed an instructional model designed based on Van Hiele's theory and supported by GSP to approach this. Considering this model, the course content was organized around the basic geometric concepts, which would be taught later to school pupils, and instructed to STs over eight weeks. Overall, the results showed a significant enhancement in STs' GCK, particularly knowledge of 2-D shapes and related concepts of perimeter and area applied within this context, in addition to a positive change in their attitudes towards technology integration, especially in the affective component, both with large effect size.

Through the model involved in this study, some difficulties related to constructing and visualizing the geometric concepts and teachers' domination of the classroom discussion, which characterize many conventional learning environments (Tay & Wonkyi, 2018), could be overcome. This, on the one side, matches previous studies findings on GSP, wherein GSP was regarded as an effective tool in constructing mental models of geometric shapes, examining relationships among these shapes, and testing related assumptions and properties (Ganesan & Eu, 2020; Meng & Sam, 2013). It, on the other side, affirms the significance of the instructional activities designed considering Van Hiele's levels. Still, these results should be explained within Hassan et al.'s (2020) view of the effectiveness of an intervention; as reported, it does not necessarily rely on the use of technology but on how such technology is employed, the knowledge domain, and the level investigated.

In that sense, the current study responds to the NCTM recommendations about implementing technology as a teaching and learning instrument in geometry classes (NCTM, 2000). It also replies to Cevikbas and Kaiser’s (2021) recent calls for more research on how mathematics teaching could be facilitated through dynamic learning environments. Accordingly, some recommendations could be presented, especially about the early usage of ICT in initial teacher education, as raised in studies like Ndlovu et al. (2013). This influences STs’ attitudes towards technology integration, which would be increased due to the high exposure to computers during their preparation program (Mangi et al., 2021; Teo, 2008).

Also, mathematics teacher educators could exploit the course of information technology more effectively by teaching STs' specific software related to mathematics (e.g., GeoGebra, GSP, Cabri 3D, Euler 3D) instead of teaching them the traditional software (e.g., Word, PowerPoint). They should also support STs to achieve different levels of employing dynamic geometry software in teaching and learning mathematics, as recommended by Zambak and Tyminski (2020).

Further to this, curriculum materials should be developed to be consistent with integrating such technologies into Van Hiele-based instruction to scaffold STs’ learning processes. This would eventually help promote STs’ TPACK and increase their attitudes towards integrating technology into their future learning since teachers usually teach in the manner they were taught (Sunzuma & Maharaj, 2020). It may also support designing professional development training to grow in-service teachers’ knowledge and attitudes to operating technology in geometry instruction.

Several limitations that might hinder generalizing the findings of this study should be considered. Firstly, the study did not extensively measure STs’ GCK due to its small number of test items adapted from TEDS-M. Second, the sample size was relatively small and was limited to STs in the second year of the mathematics teachers' preparation program at Tanat University in Egypt (a single program in a specific setting). Finally, it is necessary to take into account the constraints of conducting this research through the one-group design without including a control or comparison group, which may affect the validity of the results. Therefore, future research is needed to further explore the effectiveness of similar models in other environments.

Acknowledgments

I’m deeply grateful to the student teachers who were enrolled in the Mathematics Education program at the Faculty of Education, Tanta University, during the academic year of 2021-2022 to be a part of this research.

Declarations of Conflicting Interests

The author declares no competing interests.

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